373K Algebra I, Homework 9

From Artin

Chapter 6 (pp 221–224) 2.7, 2.8, 2.13, 3.3, 7.3, 7.4(b).

Others:

1. How many elements of order 7 must there be in a simple group of order 168?

2. Let G be a group of order 99, and $H < G$ of order 11. Prove that H is a normal subgroup.

3. Let p, q, r be distinct primes with $r > q > p$. Show that a group of rder pqr is not simple.

4. Identify a Sylow 2-subgroup of S_6.

5. Prove that there is no simple group of order 616.

6. Prove that there is no simple group of order 132.

7. (extra for entertainment) (a) Prove that two permutations in S_n are conjugate if and only if they have the same cycle type.

(b) Let $\sigma \in S_n$ be written as a product of disjoint cycles of length m_i, with each cycle of length m_i appearing k_i times (so $n = \Sigma_1^s k_i m_i$). Prove that the number of conjugates of σ is:

$$\frac{n!}{k_1!k_2!\dots k_s!m_1^{k_1}m_2^{k_2}\dots m_s^{k_s}}$$