Nyström methods for the exterior Stokes equations

O. Gonzalez

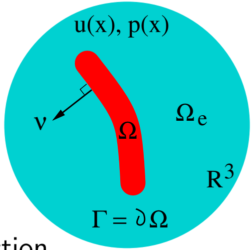
joint work with J. Li

University of Texas at Austin

Introduction

Goal. To compute the Stokes traction integral $\mathcal{F} \in \mathbb{R}$ for given data $U, \eta : \Gamma \to \mathbb{R}^3$.

$$\begin{aligned} \mu\Delta u &= \nabla p && \text{in } \Omega_e \\ \nabla\cdot u &= 0 && \text{in } \Omega_e \\ u &= U && \text{on } \Gamma \\ u, p &\to 0 && \text{as } |x| \to \infty \end{aligned}$$



$$\sigma\nu = \left[\mu(\nabla u + \nabla u^T) - pI\right]\nu \quad \text{surface traction}$$

$$\mathcal{F} = \int_\Gamma \eta\cdot\sigma\nu \; dA \quad \text{traction integral.}$$

Applications. Modeling of diffusion and transport of particles; molecular separation, mixing techniques; fluid-structure interaction.

Outline

Introduction

Formulation

Method

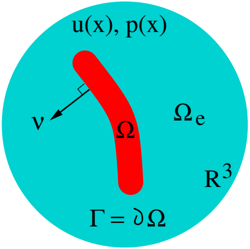
Examples

Formulation

Formulation

Setup. In dimensionless form, the equations are

$$
\begin{array}{rcll}
\Delta u & = & \nabla p & \text{in } \Omega_e \\
\nabla \cdot u & = & 0 & \text{in } \Omega_e \\
u & = & U & \text{on } \Gamma \\
u, p & \to & 0 & \text{as } |x| \to \infty
\end{array}
$$



$$\sigma\nu = \left[\nabla u + \nabla u^T - pI\right]\nu$$

$$\mathfrak{F} = \int_\Gamma \eta \cdot \sigma\nu \, dA.$$

Approach. Reformulate problem using classic single- and double-layer potentials.

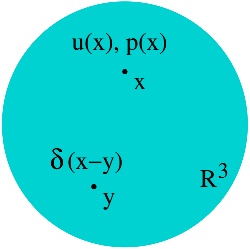
Fundamental solutions

Free-space equations.

$$\Delta u = \nabla p + f \quad \text{in } \mathbb{R}^3$$

$$\nabla \cdot u = g \quad \text{in } \mathbb{R}^3$$

$$u, p \to 0 \quad \text{as } |x| \to \infty.$$



Classic solutions.

Pt-force (Stokeslet)	$G^u(x,y), G^p(x,y)$	$f \propto \delta(x-y)$
Pt-source	$Q^u(x,y), Q^p(x,y)$	$g \propto \delta(x-y)$
Stresslet	$H^u(x,y), H^p(x,y)$	(linear combo).

Single-layer potentials

Definition. By the single-layer potentials for (u, p, σ) with density $\psi : \Gamma \to \mathbb{R}^3$ we mean

$$
\begin{aligned}
u_i(x) &= \int_\Gamma G^u_{ij}(x,y)\psi_j(y)\, dA_y \\
p(x) &= \int_\Gamma G^p_j(x,y)\psi_j(y)\, dA_y \\
\sigma_{ik}(x) &= \int_\Gamma G^\sigma_{ikj}(x,y)\psi_j(y)\, dA_y
\end{aligned}
$$

$$
G^u_{ij}(x,y) = \frac{\delta_{ij}}{r} + \frac{z_i z_j}{r^3}, \qquad G^p_j(x,y) = \frac{2z_j}{r^3}
$$

$$
G^\sigma_{ikj}(x,y) = -\frac{6 z_i z_k z_j}{r^5}, \qquad z = x - y, \qquad r = |z|.
$$

Note. They satisfy Stokes in Ω_e; representation has limited range; traction $\sigma\nu$ is *weakly-singular* on Γ.

Double-layer potentials

Definition. By the double-layer potentials for (u, p, σ) with density $\psi : \Gamma \to \mathbb{R}^3$ we mean

$$u_i(x) = \int_\Gamma H^u_{ij}(x,y)\psi_j(y)\, dA_y$$

$$p(x) = \int_\Gamma H^p_j(x,y)\psi_j(y)\, dA_y$$

$$\sigma_{ik}(x) = \int_\Gamma H^\sigma_{ikj}(x,y)\psi_j(y)\, dA_y$$

$$H^u_{ij}(x,y) = \frac{3z_i z_j\, z\cdot\nu}{r^5}, \qquad H^p_j(x,y) = -\frac{2\nu_j}{r^3} + \frac{6z_j\, z\cdot\nu}{r^5}$$

$$H^\sigma_{ikj} = \frac{2\delta_{ik}\nu_j}{r^3} + \frac{3(\delta_{ij}z_k\, z\cdot\nu + \delta_{jk}z_i\, z\cdot\nu + \nu_i z_j z_k + \nu_k z_i z_j)}{r^5} - \frac{30 z_i z_j z_k\, z\cdot\nu}{r^7}.$$

Note. They satisfy Stokes in Ω_e; representation has limited range; traction $\sigma\nu$ is *hyper-singular* on Γ.

Proposed formulation

Description. For the exterior problem, we propose a mixed-layer formulation with a parallel surface.

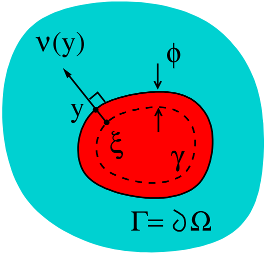
$$u(x) = \lambda \int_{\gamma} G^u(x,\xi)\psi(y(\xi))\, dA_\xi + (1-\lambda)\int_{\Gamma} H^u(x,y)\psi(y)\, dA_y$$

$p(x), \sigma(x)$ represented similarly

$0 < \lambda < 1$ interpolation parameter

γ parallel surface

$0 < \phi < \phi_\Gamma$ offset parameter.



Note. Interpolation leads to a complete range; parallel surface is convenient for numerics.

Boundary integral equation

For given data $U : \Gamma \to \mathbb{R}^3$, the density $\psi : \Gamma \to \mathbb{R}^3$ is determined by the condition

$$\lim_{\substack{x_o \to x \\ x_o \in \Omega^e}} u(x_o) = U(x), \ \forall x \in \Gamma.$$

Equivalently

$$\mathcal{G}\psi + \mathcal{H}\psi + c\psi = U$$

where

$$c = 2\pi(1 - \lambda)$$

$$\mathcal{G}\psi(x) = \int_\gamma G(x, \xi)\psi(y(\xi))\, dA_\xi, \qquad G(x, \xi) = \lambda G^u(x, \xi)$$

$$\mathcal{H}\psi(x) = \int_\Gamma H(x, y)\psi(y)\, dA_y, \qquad H(x, y) = (1 - \lambda)H^u(x, y).$$

Solvability result

Problem. Given $U : \Gamma \to \mathbb{R}^3$, we seek $\psi : \Gamma \to \mathbb{R}^3$ such that

$$\mathcal{G}\psi + \mathcal{H}\psi + c\psi = U.$$

Solvability result

Problem. Given $U : \Gamma \to \mathbb{R}^3$, we seek $\psi : \Gamma \to \mathbb{R}^3$ such that

$$\mathcal{G}\psi + \mathcal{H}\psi + c\psi = U.$$

Theorem (Hebeker). There exists a unique density $\psi \in C^0$ for any closed, bounded Lyapunov surface $\Gamma \in C^{1,1}$, offset parameter $\phi \in (0, \phi_\Gamma)$, interpolation parameter $\lambda \in (0, 1)$ and data $U \in C^0$.

Traction result

Problem. Given any vector field $\eta : \Gamma \to \mathbb{R}^3$, we seek to compute the traction integral

$$\mathcal{F} = \int_\Gamma \eta \cdot \sigma^e \nu \, dA$$

where

$$\sigma^e(x) = \lim_{\substack{x_o \to x \\ x_o \in \Omega^e}} \sigma(x_o).$$

Traction result

Problem. Given any vector field $\eta : \Gamma \to \mathbb{R}^3$, we seek to compute the traction integral

$$\mathcal{F} = \int_\Gamma \eta \cdot \sigma^e \nu \, dA$$

where

$$\sigma^e(x) = \lim_{\substack{x_o \to x \\ x_o \in \Omega^e}} \sigma(x_o).$$

Theorem. If $\Gamma \in C^{1,1}$, $\psi \in C^{0,1}$ and $\eta \in C^{0,\alpha}$ $(\alpha > 0)$, then $\mathcal{F}$ exists and can be expressed as a weakly-singular integral depending only pointwise on ψ and η, namely

$$\mathcal{F} = \int_\Gamma \int_\Gamma K[\eta, \psi](x, y) \, dA_x dA_y.$$

Traction integral kernel

$$
\begin{aligned}
K[\eta,\psi](x,y) &= \eta_i(x)G_{ikj}(x,y)\psi_j(y)\nu_k(x) \\
&+\tfrac{1}{2}\widehat{\eta}_i(x,y)H_{ikjl}(x,y)\widehat{\psi}_j(y,x)\nu_l(y)\nu_k(x) \\
&+\tfrac{1}{2}\widehat{\eta}_i(x,y)H_{ikjl}(x,y)\psi_j(x)\widehat{\nu}_l(y,x)\nu_k(x) \\
&+\tfrac{1}{2}\widehat{\eta}_i(x,y)H_{ikjl}(x,y)\psi_j(x)\nu_l(x)\widehat{\nu}_k(x,y)
\end{aligned}
$$

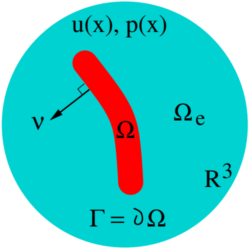
$$
G = \lambda J_\phi G^\sigma, \quad H = (1-\lambda)H^\sigma, \quad \widehat{g}(x,y) = g(x) - g(y).
$$

Method

Method

Setup. Given $U, \eta : \Gamma \to \mathbb{R}^3$, we seek to compute $\mathcal{F} \in \mathbb{R}$.

$$\begin{aligned} \Delta u &= \nabla p && \text{in } \Omega_e \\ \nabla \cdot u &= 0 && \text{in } \Omega_e \\ u &= U && \text{on } \Gamma \\ u, p &\to 0 && \text{as } |x| \to \infty \end{aligned}$$



$$\sigma\nu = \left[\nabla u + \nabla u^T - pI\right]\nu$$

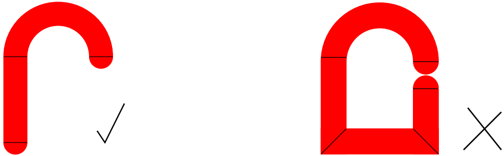
$$\mathcal{F} = \int_\Gamma \eta \cdot \sigma\nu \, dA.$$

Approach. Consider Nyström-type method on BIE formulation.

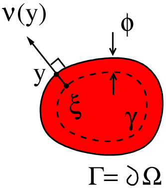
$$\mathcal{G}\psi + \mathcal{H}\psi + c\psi = U, \qquad \mathcal{F} = \int_\Gamma \int_\Gamma K[\eta, \psi](x, y) \, dA_x dA_y.$$

Surface, kernels

Assumption (A0). $\Gamma \in C^{m+1,1}$, $m \geq 0$, closed, bdd, Lyapunov.



Assumption (A1). $\mathcal{G}\psi(x) = \int_\gamma G(x,\xi)\psi(y(\xi))dA_\xi$. G class $C^{m,1}$.



$$y = \xi + \phi\nu(\xi)$$

$$\nu(y) = \nu(\xi).$$

Surface, kernels

Assumption (A2). $\mathcal{H}\psi(x) = \int_\Gamma H(x,y)\psi(y)\, dA_y$. H weakly-singular; class $C^{m,1}$ away from diagonal. Moreover

$$H(x,y) = \frac{g(x,y)}{|x-y|}, \quad \forall x \neq y$$

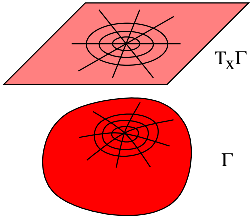
$g(x,y)$ bounded; class C^{m+1} in x; class C^m in y

$$|D_x^\alpha g|, |D_y^\alpha g| \leq \frac{C}{|x-y|^{|\alpha|}}, \quad \forall x \neq y$$

D_x, D_y are surface derivatives; α is multi-index.

Surface, kernels

Assumption (A3). For each $x \in \Gamma$ we assume



(i) $g(x,y)|_{y=y(\rho,\theta)}$ class $C^{0,1}$ in (ρ,θ)

(ii) $\displaystyle \lim_{\substack{\rho\to 0\\ \theta \text{ fixed}}} g(x,y) = \lim_{\substack{\rho\to 0\\ \theta+\pi \text{ fixed}}} g(x,y)$

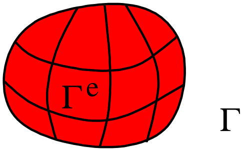
(iii) $\displaystyle g(x,y) - \lim_{\substack{\rho\to 0\\ \theta(y) \text{ fixed}}} g(x,y)$ class $C^{0,1}$ in y.

Mesh

Assumption (A4). We consider a decomposition of Γ into non-overlapping quadrature elements Γ^e, $e = 1 \ldots E$. We assume

$$Ch^2 \leq |\Gamma^e| \leq C' h^2, \quad \forall e, E$$

$$h = \max_e \{\text{diam}(\Gamma^e)\}.$$



Quadrature rule

Assumption (A5). In each Γ^e we introduce nodes x_q^e and weights $W_q^e > 0$, $q = 1 \dots Q$, such that

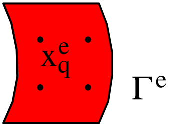
$$\int_\Gamma f(x)\, dA_x = \sum_e \int_{\Gamma^e} f(x)\, dA_x \doteq \sum_e \sum_q f(x_q^e) W_q^e .$$

We assume

$$Ch^2 \leq \textstyle\sum_q W_q^e \leq C' h^2$$

$$Ch \leq \operatorname{dist}(x_q^e, \partial\Gamma^e) \leq h \qquad \forall q, e, E.$$

$$Ch \leq \min_{\substack{e\neq e' \\ q\neq q'}} |x_q^e - x_{q'}^{e'}| \leq h$$



Quadrature error

Assumption (A6). We consider quadrature errors defined by

$$\tau(e, f, h) = \frac{1}{|\Gamma^e|} \left| \int_{\Gamma^e} f(x)\, dA_x - \sum_q f(x_q^e) W_q^e \right|.$$

We assume

$$\max_e \tau(e, f, h) \to 0 \quad \text{as} \quad h \to 0, \quad \forall f \in C^0$$

$$\max_e \tau(e, f, h) \leq C_f h^\ell, \quad \forall f \in C^{\ell-1,1}$$

$\ell \geq 1$ is order of quadrature rule.

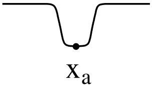
Nodal partition of unity functions

Assumption (A7). To each node x_a, $a = (q, e)$ we associate functions $\zeta_a, \widehat{\zeta}_a \in C^0$. We assume

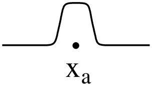
$$\zeta_a(x), \widehat{\zeta}_a(x) \in [0,1], \quad \zeta_a(x) + \widehat{\zeta}_a(x) = 1, \quad \forall x \in \Gamma$$

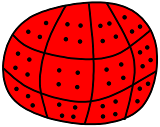
$$\zeta_a(x) \leq \frac{C|x - x_a|^2}{h^2}, \quad \text{diam}(\text{supp}(\widehat{\zeta}_a)) \leq Ch.$$

$\zeta_a(x)$



$\widehat{\zeta}_a(x)$

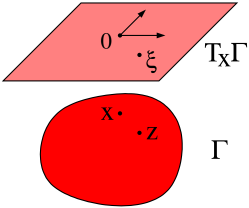




Γ

Local polynomials

Definition. In a neighborhood of any given $x \in \Gamma$ we can define a local poly $R_x(z)$ of any given degree.



$\xi_x(z)$ is projection of $z - x$ onto $T_x\Gamma$

$$R_x(z) = C_x + C_x^{\alpha}\,\xi_x^{\alpha}(z) + C_x^{\alpha\beta}\,\xi_x^{\alpha}(z)\,\xi_x^{\beta}(z) + \cdots$$

Note. $R_x(z)$ is well-defined in a neighborhood of x; there is a uniform bound on the size of this neighborhood.

Discretized operators

Definition. For any given degree $p \geq 0$ we define discretized operators by

$$\mathcal{G}_h\psi(x) = \sum_b G(x, x_b)\psi(x_b)W_b$$

$$\mathcal{H}_h\psi(x) = \sum_b \left[\zeta_b(x)H(x, x_b)\psi(x_b)W_b + \widehat{\zeta}_b(x)R_x(x_b)\psi(x_b)\right]$$

where $R_x(z)$ is a local poly at x of degree p defined such that

$$\mathcal{H}_h f(x) = \mathcal{H} f(x) \quad \text{for all local polys } f \text{ at } x \text{ of degree } \leq p.$$

Assumption (A8). The coeffs of $R_x(z)$ are unique and bounded for all $h > 0$.

Discretized equation

Given data U we seek an approximate density ψ_h such that

$$\mathcal{G}_h\psi_h(x) + \mathcal{H}_h\psi_h(x) + c\psi_h(x) = U(x), \quad \forall x \in \Gamma.$$

Any solution ψ_h can be constructed in two steps

(i) Solve nodal equations

$$\mathcal{G}_h\psi_h(x_a) + \mathcal{H}_h\psi_h(x_a) + c\psi_h(x_a) = U(x_a), \quad \forall a.$$

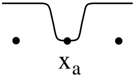
(ii) Interpolate nodal values

$$\psi_h(x) = \frac{1}{c}\Big[U(x) - \mathcal{G}_h\psi_h(x) - \mathcal{H}_h\psi_h(x)\Big], \quad \forall x \in \Gamma.$$

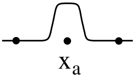
Illustration: method with $p = 0$

Partition of unity. We can choose $\zeta_a(x)$ and $\widehat{\zeta}_a(x)$ such that

$$\zeta_a(x_b) = 1 - \delta_{ab}$$



$$\widehat{\zeta}_a(x_b) = \delta_{ab}$$



Local polynomial. $R_x(z) = C_x$ for each $x \in \Gamma$. Moment condition gives

$$C_x = \frac{\int_\Gamma H(x,y)\, dA_y - \sum_b \zeta_b(x) H(x, x_b) W_b}{\sum_b \widehat{\zeta}_b(x)}.$$

Illustration: method with $p = 0$

Nodal equations. For each node x_a we get

$$\sum_b G(x_a, x_b)\psi_h(x_b)W_b + \sum_{b\neq a} H(x_a, x_b)\Big[\psi_h(x_b) - \psi_h(x_a)\Big]W_b = U(x_a).$$

Interpolation equation. At any x we have

$$\psi_h(x) = \frac{1}{c}\Bigg[U(x) - \sum_b G(x, x_b)\psi_h(x_b)W_b - \sum_b \zeta_b(x)H(x, x_b)\psi_h(x_b)W_b - \sum_b \widehat{\zeta}_b(x)C_x\psi_h(x_b)\Bigg].$$

Solvability, convergence result

Setup. Consider method with a quad rule of order $\ell \geq 1$ and a local poly of degree $p \geq 0$. The original and discretized eqs are

$$\mathcal{G}\psi + \mathcal{H}\psi + c\psi = U \tag{1}$$

$$\mathcal{G}_h\psi_h + \mathcal{H}_h\psi_h + c\psi_h = U. \tag{2}$$

Solvability, convergence result

Setup. Consider method with a quad rule of order $\ell \geq 1$ and a local poly of degree $p \geq 0$. The original and discretized eqs are

$$\mathcal{G}\psi + \mathcal{H}\psi + c\psi = U \tag{1}$$

$$\mathcal{G}_h\psi_h + \mathcal{H}_h\psi_h + c\psi_h = U. \tag{2}$$

Theorem. Assume (A0)–(A8). If (1) has a unique solution, then so does (2) for all $h > 0$ sufficiently small. If $\Gamma \in C^{m+1,1}$ and $\psi \in C^{m,1}$, then as $h \to 0$

$$\begin{aligned} ||\psi - \psi_h||_\infty &\to 0 && \forall \ell \geq 1, p \geq 0, m \geq 0 \\ ||\psi - \psi_h||_\infty &\leq Ch && \forall \ell \geq 1, p = 0, m \geq 1 \\ ||\psi - \psi_h||_\infty &\leq Ch^{\min(\ell,p,m)} && \forall \ell \geq 1, p \geq 1, m \geq 1. \end{aligned}$$

Idea of proof

Sketch. Pf is based on theory of collectively cmpct ops on C^0. We show

(i) $\mathcal{G}_h f(x)$, $\mathcal{H}_h f(x)$ equibounded for given f.

(ii) $\mathcal{G}_h f(x)$, $\mathcal{H}_h f(x)$ equicontinuous for given f.

(iii) $\mathcal{G}_h f \to \mathcal{G} f$, $\mathcal{H}_h f \to \mathcal{H} f$ in C^0 for given f.

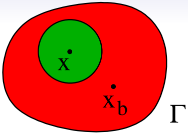
We estimate

(iv) $||\mathcal{G} f - \mathcal{G}_h f||_\infty$ and $||\mathcal{H} f - \mathcal{H}_h f||_\infty$ for given f.

Then by collective compactness we have

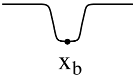
$$||\psi - \psi_h||_\infty \leq C||(\mathcal{G} - \mathcal{G}_h)\psi + (\mathcal{H} - \mathcal{H}_h)\psi||_\infty.$$

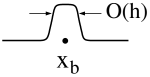
Idea of proof



Equiboundedness of $\mathcal{H}_h$.

$f \in C^0$, $x \in \Gamma$ given.

$\zeta_b(x)$ 

$\widehat{\zeta}_b(x)$ 

Green patch at x of size $O(h)$; contains finite # quad elements.

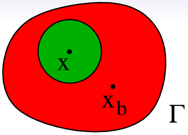
$$\mathcal{H}_h f(x) = \underbrace{\sum_b}_{\text{green+red}} \zeta_b(x) H(x, x_b) f(x_b) W_b + \underbrace{\sum_b}_{\text{green}} \widehat{\zeta}_b(x) R_x(x_b) f(x_b).$$

$$\left| \sum_{\text{green}} \zeta_b(x) H(x, x_b) f(x_b) W_b \right| \leq \frac{C|x - x_b|^2}{h^2} \frac{|g(x,y)|}{|x - x_b|} |f(x_b)|\, h^2 \leq C||f||.$$

$$\left| \sum_{\text{green}} \widehat{\zeta}_b(x) R_x(x_b) f(x_b) \right| \leq C\,(C + Ch + \cdots + Ch^p)\, |f(x_b)| \leq C||f||.$$

Idea of proof

Equiboundedness of $\mathcal{H}_h$.



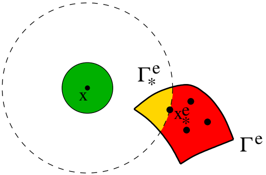
$$\Big|\sum_{\text{red}} \zeta_b(x) H(x,x_b) f(x_b) W_b\Big|$$

$$\leq \underbrace{\sum_e \sum_q}_{\text{red}} \frac{|g(x,x_q^e)|}{|x-x_q^e|} |f(x_q^e)|\, W_q^e$$

$$\leq \sum_{e\in\text{red}} \frac{C}{|x-x_*^e|} ||f|| \Big(\sum_q W_q^e\Big)$$

$$\leq \sum_{e\in\text{red}} \frac{C|\Gamma_*^e|}{|x-x_*^e|} ||f||$$

$$\leq \sum_{e\in\text{red}} C \int_{\Gamma_*^e} \frac{1}{|x-y|}\, dA_y\, ||f|| \qquad \leq C||f||.$$



$$C|\Gamma^e| \leq |\Gamma_*^e| \leq |\Gamma^e|$$

Idea of proof

Estimate of $||\mathcal{H}f - \mathcal{H}_h f||$

$f \in C^{m,1}$, $x \in \Gamma$ given.

Quadrature rule of order ℓ given.

$T_x^p f$ local Taylor poly at x of degree p.

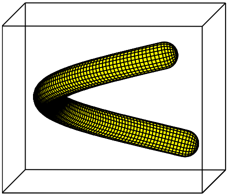
$\mathcal{H}$,$\mathcal{H}_h$ equal on local polys at x of degree p.

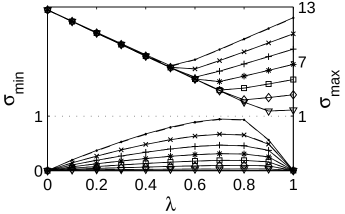
$$\begin{aligned} |\mathcal{H}f(x) - \mathcal{H}_h f(x)| &= |\mathcal{H}(f - T_x^p f)(x) - \mathcal{H}_h(f - T_x^p f)(x)| \\ &\leq Ch^{\min(\ell,p,m)}, \qquad \forall \ell \geq 1, p \geq 1, m \geq 1 \\ &\qquad\qquad \text{uniformly in } x. \end{aligned}$$

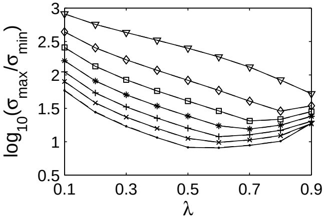
$$\Big[p = 0 \quad \& \quad \text{anti-podal condition}\Big] \doteq \Big[p = 1 \quad \& \quad O(h) \text{ terms}\Big].$$

Examples

Method with $p = 0$, $\ell = 1$: conditioning vs λ, ϕ







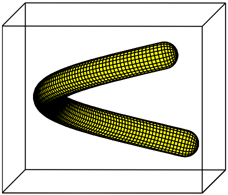
$\lambda \in (0, 1)$ interpolation param

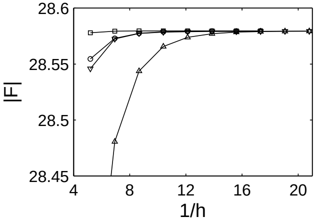
$\phi \in (0, \phi_\Gamma)$ offset surface param

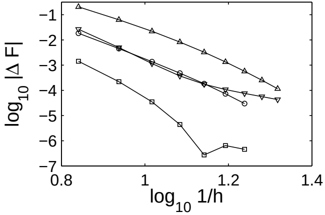
$\phi/\phi_\Gamma = \frac{1}{8}$ (dots), $\frac{2}{8}$ (crosses), $\frac{3}{8}$ (pluses), ..., $\frac{7}{8}$ (triangles)

Condition number $\frac{\sigma_{\max}}{\sigma_{\min}} \leq 10^{\mathbf{1.5}}$ for $(\lambda, \phi/\phi_\Gamma)$ near $(\frac{1}{2}, \frac{1}{2})$

Method with $p = 0$: traction integral accuracy vs h







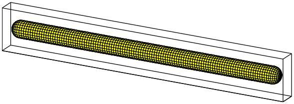
BC: rotation about horizontal

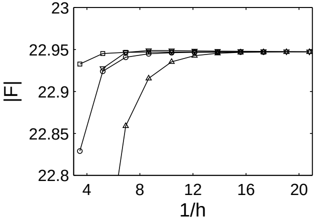
Traction integral: total force

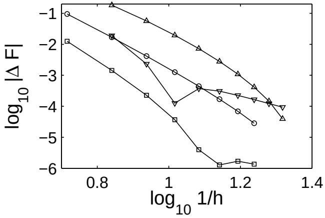
Results for $\ell = 1, 2$ and various λ, ϕ

Convergence is visible; limited by iterative solver; apparent rate is > 1

Method with $p = 0$: traction integral accuracy vs h







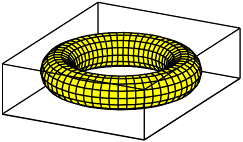
BC: transverse translation

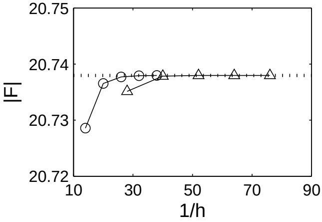
Traction integral: total force

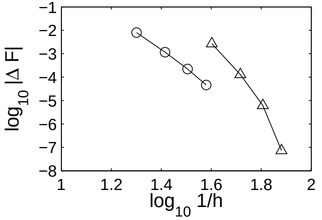
Results for $\ell = 1, 2$ and various λ, ϕ

Convergence is visible; limited by iterative solver; apparent rate is > 1

Method with $p = 0$: traction integral accuracy vs h







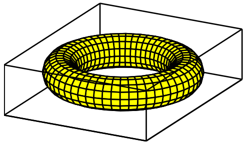
BC: translation along z-axis

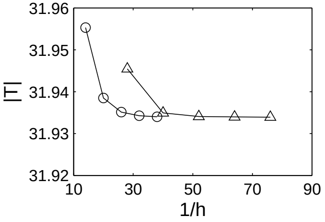
Traction integral: force along z-axis

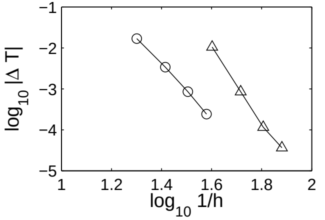
Results for $\ell = 1, 2$ and $\lambda = 1/2$, $\phi/\phi_\Gamma = 1/2$

Convergence is visible; apparent rate is > 1

Method with $p = 0$: traction integral accuracy vs h







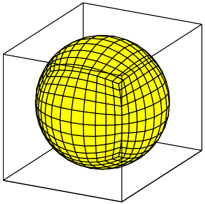
BC: rotation about z-axis

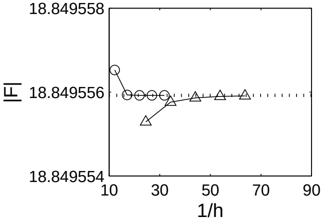
Traction integral: torque about z-axis

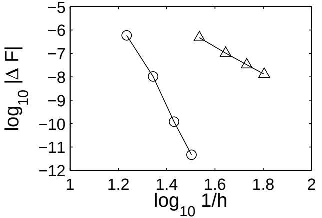
Results for $\ell = 1, 2$ and $\lambda = 1/2$, $\phi/\phi_\Gamma = 1/2$

Convergence is visible; apparent rate is > 1

Method with $p = 0$: traction integral accuracy vs h







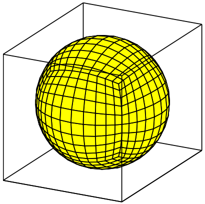
BC: translation

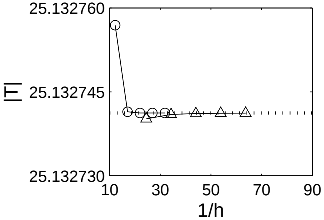
Traction integral: total force

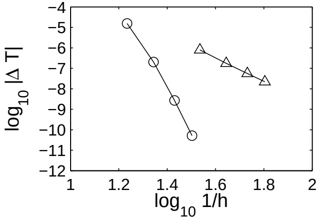
Results for $\ell = 1, 2$ and $\lambda = 1/2$, $\phi/\phi_\Gamma = 1/2$

Convergence is visible; apparent rate is > 1

Method with $p = 0$: traction integral accuracy vs h







BC: rotation

Traction integral: total torque

Results for $\ell = 1, 2$ and $\lambda = 1/2$, $\phi/\phi_\Gamma = 1/2$

Convergence is visible; apparent rate is > 1

Summary

- Mixed-layer formulation is complete for exterior Stokes.

- Traction integral has nice characterization.

- Nyström w/local polynomial is provably convergent.

- Lowest-order method is extremely simple.

- Open quad rule, anti-podal symmetry seem important.

preprints available at og@math.utexas.edu