M346 Practice Second Exam
Originally given November, 2, 2000

Problem 1: Find all the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & -3 & 5 \end{pmatrix}.$$

Problem 2: Find a matrix with eigenvalues 1, 2 and 3 and corresponding eigenvectors $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$,

Problem 3: The eigenvalues of the matrix $A = \begin{pmatrix} 0 & 3 & 0 \\ 3 & 0 & 4 \\ 0 & 4 & 0 \end{pmatrix}$ are 5, 0, and -5.

a) Find the eigenvectors.

b) Decompose the vector $(50, 0, 0)^T$ as a linear combination of eigenvectors.

c) Solve the differential equation $d\mathbf{x}/dt = A\mathbf{x}$ with initial condition $\mathbf{x}(0) = (50, 0, 0)^T$.

Problem 4: Consider discrete-time evolution equations

$$\begin{aligned} x_1(n) =& x_1(n-1) + 2x_2(n-1) \\ x_2(n) =& x_1(n-1) + 3x_2(n-1). \end{aligned}$$

a) How many stable modes does this system have? How many neutrally stable modes? How many unstable modes?

b) Write down the general solution to this system of equations.

c) Describe qualitatively the behavior of $\mathbf{x}(n)$ for large n (both size and direction), given typical initial conditions.

Problem 5: Consider the nonlinear system of differential equations

$$\begin{aligned} \frac{dx_1}{dt} =& x_1(3 - x_1 - 2x_2) \\ \frac{dx_2}{dt} =& x_2(2 - x_1 - x_2) \end{aligned}$$

a) Find the fixed points. [There are four of them]

b) For each fixed point, find a linear system of equations that approximates the dynamics near the fixed point.

c) Which (if any) of the fixed points are stable?