

# PRELIMINARY EXAMINATION IN TOPOLOGY

August 1999      4 Hours

**Instructions:** Solve two of the three problems in each part.

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## Part I.

1. Compute the fundamental group and all homology groups of the spaces  $X, Y$  and  $Z$  below:
  - (a)  $X$  is obtained by attaching a 2-cell  $D$  to a circle  $S$  by a map of degree 2 from the boundary circle of  $D$  to  $S$ .
  - (b)  $Y$  is obtained from  $X$  by attaching another 2-cell  $D'$  by a map of degree 4 from the boundary circle of  $D'$  to  $S$ .
  - (c)  $Z$  is obtained from  $Y$  by attaching yet another 2-cell  $D''$  by a map of degree 5 from the boundary circle of  $D''$  to  $S$ .
2. Sketch pictures of the universal covering spaces of each of the following:
  - (a)  $\mathbb{RP}(2) \vee \mathbb{RP}(2)$
  - (b)  $S^1 \vee \mathbb{RP}(2)$
3. Give a proof or example to settle each of the following questions:
  - (a) Does every self-map of  $\mathbb{RP}(2) \vee \mathbb{RP}(2) \vee \mathbb{RP}(2) \vee \mathbb{RP}(2)$  have a fixed point?
  - (b) Does every self map of  $\mathbb{RP}(2) \# \mathbb{RP}(2) \# \mathbb{RP}(2) \# \mathbb{RP}(2)$  have a fixed point?

**Part II.** All manifolds in this part are assumed to be smooth, compact and without boundary.

4. Let  $X$  be a connected, oriented 2-manifold, and let  $f : S^1 \rightarrow X$  be a smooth embedding of a circle. We say that  $f$  is *separating* if  $X - f(S^1)$  is disconnected; otherwise  $f$  is *nonseparating*.
  - (a) For closed 1-forms  $\eta$  on  $F$ , what are the possible values of  $\int_{S^1} f^*\eta$  if  $f$  is separating? Why?
  - (b) Does the answer remain the same if  $f$  is nonseparating? Prove or find a counterexample.
5. Let  $f : X \rightarrow Y$  be a smooth map between manifolds, and let  $Z \subset X$  be a submanifold with  $\dim Z = \dim Y$  and mod 2 degree  $\deg_2(f|_Z) \neq 0$ . Must there be a compact submanifold  $W \subset X$  whose mod 2 intersection number  $I_2(W, Z)$  is nonzero? Why?
6. Let  $X$  be an oriented manifold whose Euler characteristic  $\chi(X)$  is nonzero. Prove that there is no submersion  $f : X \rightarrow S^1$ . (*Hint:* Vector fields)