1. Let $f \in L^\infty(\mu)$ be a nonnegative bounded $\mu$-measurable function. Consider the set $R_f$ consisting of all positive real numbers $w$ such that $\mu(\{x : |f(x) - w| \leq \varepsilon\}) > 0$ for every $\varepsilon > 0$.
   
   (a) Prove that $R_f$ is compact.
   
   (b) Prove that $\|f\|_{L^\infty} = \sup R_f$.

2. Let $f, f_1, f_2, \ldots$ be functions in $L^1([0,1])$ such that $f_k \to f$ pointwise almost everywhere. Show that $\|f_k - f\|_1 \to 0$ if and only if for every $\varepsilon > 0$ there exists $\delta > 0$, such that $|\int_A f_k| < \varepsilon$ for all $k$ and all measurable set $A \subset [0,1]$ with measure $|A| < \delta$.

3. Let $p > 0$, and denote by $L^p_{\text{weak}}(\mathbb{R})$ the space of all measurable functions $f : \mathbb{R} \to \mathbb{R}$ for which
   
   $$N_p(f) = \sup_{\alpha > 0} \alpha^p |\{x \in \mathbb{R}^n : |f(x)| > \alpha\}|$$
   
   is finite. Prove that the simple functions are not dense in $L^p_{\text{weak}}(\mathbb{R})$, in the sense that there exists a function $f \in L^p_{\text{weak}}(\mathbb{R})$ such that $N_p(f - h_k) \to 0$ fails to hold for every sequence of simple functions $h_1, h_2, \ldots$.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be absolutely continuous with compact support, and let $g \in L^1(\mathbb{R})$. Prove that $f * g$ is absolutely continuous on $\mathbb{R}$. 