1. Consider a real \( m \times n \) matrix \( A \).

(a) Describe how singular value decomposition can be used to solve the least squares problem \( \min_i \|Ax - b\| \).

(b) Describe two methods for computing the vector \( u \) that minimizes,

\[
\min_{u \neq 0} \frac{\langle u, Au \rangle}{\langle u, u \rangle}.
\]

2. Let \( f(x) : \mathbb{R}^n \rightarrow \mathbb{R}^n \) be a smooth function such that \( f(a) = 0 \) and the matrix \( \frac{\partial f}{\partial x} \) is full rank.

(a) Write down the Newton method for solving \( f(x) = 0 \).

(b) Show that the Newton method converges to the root \( a \) if the initial condition \( x_0 \) is sufficiently close to \( a \).

(c) Suppose that the following iteration is used instead

\[
x_{n+1} = x_n - A f(x_n),
\]

where \( A \) is a \( n \times n \) matrix. Find a sufficient condition on \( A \) such that this iteration is also convergent for \( x_0 \) close to \( a \).

3. In the numerical integration formula

\[
\int_{-1}^{1} f(x) \, dx = af(-1) + bf(c),
\]

(a) if constants \( a, b, c \) can be chosen arbitrarily, what is the highest degree \( k \) such that this formula is exact for all polynomials of degree up to \( k \)?

(b) Find the constants \( a, b, c \) for which the formula is exact for all polynomials of degree up \( k \).