PRELIMINARY EXAM: ALGEBRAIC TOPOLOGY

Date: January 9, 2014
Instructions: Do all three problems.
Time Limit: 90 minutes

Problem 1. Let $F$ be the closed surface obtained by gluing together two copies of the Möbius band by a homeomorphism between their boundaries.

(a) Identify the surface $F$.
(b) Let $A \subset F$ be the image of the boundaries of the Möbius bands. Compute the relative homology $H_*(F, A)$.

Problem 2. Let $X$ be the 1-skeleton of a tetrahedron and $Y$ the 1-skeleton of a cube. Is $Y$ a covering space of $X$? (If “yes”, exhibit a covering, and if “no”, prove that no such covering exists.)

Problem 3.
(a) Classify the maps $S^1 \to \mathbb{R}P^1$ up to homotopy.
(b) Let $f : S^1 \to \mathbb{R}P^1 \subset \mathbb{R}P^3$ be a map and let $X_f$ be the space obtained by attaching a 2-cell to $\mathbb{R}P^3$ via $f$. Compute $\pi_1(X_f)$. 