Problem 1

Find a function $f$ such that $f$ is holomorphic in $\mathbb{C} \setminus \mathbb{N}$, and that at each positive integer $n$, $f$ has a pole of order $n$.

Problem 2

Suppose that $f$ is a meromorphic function on the punctured disk $0 < |z| < 1$, having poles at $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$. Show that for every $r > 0$, the restriction of $f$ to $0 < |z| < r$ has a range that is dense in $\mathbb{C}$.

Problem 3

Suppose $\Omega \subset \mathbb{C}$ is bounded, $\{f_n\}$ is a sequence of continuous functions on $\overline{\Omega}$ that are holomorphic in $\Omega$ and $\{f_n\}$ converges uniformly on the boundary of $\Omega$. Prove that $\{f_n\}$ converges uniformly on $\Omega$.

Problem 4

Assume that $f$ and $g$ are holomorphic functions in an open and connected region $\Omega$ such that $|f(z)| + |g(z)|$ is constant for all $z \in \Omega$. Prove that then, $f$ and $g$ must both be constant.

Problem 5

Let $(f_1, f_2, \ldots)$ be a sequence of analytic functions on a connected open domain $\Omega$, converging to a non-constant function $f$, uniformly on compact subsets of $\Omega$. Show that if each $f_n$ is one-to-one, then so is $f$. 