1. Describe the subgroup of $S_n$ that fixes the polynomial $x_1 + x_2$ under the standard action on $\mathbb{Q}[x_1, \ldots, x_n]$ obtained by permuting variables. Use this to give an upper bound for the degree over $\mathbb{Q}$ of the real part of $z \in \mathbb{C}$, if $z$ is algebraic of degree $n$ over $\mathbb{Q}$. Give a condition under which the bound is sharp. (Here $\mathbb{Q}, \mathbb{C}$ are the fields of rational and complex numbers, respectively).

2. Let $K$ be a field and $f(x) \in K[x]$ be a separable, irreducible polynomial of degree 5. If $a, b$ are distinct roots of $f(x)$ with $K(a) = K(b)$, show that $K(a)/K$ is Galois.

3. Let $K/\mathbb{Q}$ be an extension of degree $n$, where $\mathbb{Q}$ is the field of rational numbers. Show that the number of subfields of $K$ is at most $2^n$. Suppose that $K = \mathbb{Q}(\alpha, \beta)$ and prove that there exists $m, 0 \leq m \leq 2^n$ such that $K = \mathbb{Q}(\alpha + m\beta)$. 