Do two of the three questions below. Please indicate which questions you want graded.

1. Let $R$ be an integral domain such that every prime ideal of $R$ is principal.
   
   a) Consider the set of ideals of $R$ which are not principal. Prove that if this set is non-empty, then it contains an element $I$ which is maximal under inclusion.

   b) Prove that $R$ is a principal ideal domain. (Hint: Show that $I$ is principal.)

2. Let $R$ be a PID, $\pi$ an irreducible element of $R$ and consider the subset $M$ of $R^2$ of pairs $(x, y)$ with $\pi$ dividing $y$ and $\pi^2$ dividing $y - x\pi$. Show that $M$ is a submodule of $R^2$ of rank 2 and find a basis $v_1, v_2$ of $R^2$ and $a_1, a_2 \in R$ with $a_1$ dividing $a_2$ such that $a_1v_1, a_2v_2$ is a basis of $M$.

3. Let $H$ be a subgroup of a finite group $G$. Define $N_G(H) = \{g \in G \mid gHg^{-1} = H\}$. Prove that $N_G(H)$ is a subgroup of $G$ and that $H$ is normal subgroup of $N_G(H)$. Let $G = SL_2(k)$ the group of $2 \times 2$ matrices of determinant one over a field $k$. Let $H$ be the subgroup of $G$ consisting of upper triangular matrices with ones along the diagonal. Compute $N_G(H)$. 