1. Prove the Mazur Separation Theorem: Let \( X \) be an NLS, \( Y \) a linear subspace of \( X \), and \( w \in X, w \notin Y \). If \( d = \text{dist}(w, Y) = \inf_{y \in Y} \|w - y\|_X > 0 \), then there exists \( f \in X^* \) such that \( \|f\|_{X^*} \leq 1, f(w) = d \), and \( f(y) = 0 \) for all \( y \in Y \).

2. Let \( X \) be a vector space and let \( W \) be a vector space of linear functionals on \( X \). Suppose that \( W \) separates points of \( X \), meaning that for any \( x, y \in X, x \neq y \), there exists \( w \in W \) such that \( w(x) \neq w(y) \). Let \( X \) be endowed with the smallest topology such that each \( w \in W \) is continuous (we call this the \( W \)-weak topology of \( X \)).
   
   (a) Describe a \( W \)-weak open set of 0.
   
   (b) Prove that if \( L \) is a \( W \)-weakly continuous linear functional on \( X \), then \( L \in W \). [Hint: Consider the inverse image of \( B_1(0) \subset \mathbb{F} \), which must contain a \( W \)-weak open set of 0, and apply the result from linear algebra that if \( w_i, i = 1, 2, ..., n \), and \( L \) are linear functionals on \( X \) such that \( L(x) = 0 \) whenever \( w_i(x) = 0 \) for all \( i \), then \( L \) is a linear combination of the \( w_i \).]
   
   (c) Based on this result, if \( X \) is an NLS, characterize the set of weak-* continuous linear functionals on \( X^* \).

3. Let \( \Omega = (-1, 1)^2 \subset \mathbb{R}^2 \) and \( T : D(\Omega) \rightarrow D(-1, 1) \) be defined by \( T\varphi(x, y) = \varphi(x, 0) \).
   
   (a) Show that \( T \) is a (sequentially) continuous linear operator.
   
   (b) Note that \( T' : D'(-1, 1) \rightarrow D'(\Omega) \). Determine \( T'(\delta_0) \) and \( T'(\delta_0') \), where \( \delta_0 \) is the usual Dirac point distribution in one space dimension at 0.