1. For $f \in L^1(\mathbb{R})$ denote by $Mf$ be the restricted maximal function defined by

$$(Mf)(x) = \sup_{0 < t < 1} \frac{1}{2t} \int_{x-t}^{x+t} |f(z)| \, dz.$$ 

Show that $M(f * g) \leq (Mf) * Mg$ for all $f, g \in L^1(\mathbb{R})$.

2. Let $1 \leq p, q \leq \infty$ with $\frac{1}{p} + \frac{1}{q} = 1$. Show that if $f \in L^p(\mathbb{R}^n)$ and $g \in L^q(\mathbb{R}^n)$ then $f * g$ is bounded and continuous on $\mathbb{R}^n$.

3. Let $B$ be the closed unit ball in $\mathbb{R}^n$, and let $f_1, f_2, f_3, \ldots$ be nonnegative integrable functions on $B$. Assume that

(i) $f_k \to f$ almost everywhere.

(ii) For every $\varepsilon > 0$ there exists $M > 0$ such that

$$\int_{\{x \in B : f_k(x) > M\}} f_k(x) \, dx < \varepsilon, \quad k = 1, 2, 3, \ldots$$

Show that $f_k \to f$ in $L^1(B)$.

4. Let $f, f_1, f_2, \ldots$ be increasing functions on $[a, b]$. If $\sum_k f_k$ converges pointwise to $f$ on $[a, b]$, show that $\sum_k f_k'$ converges to $f'$ almost everywhere on $[a, b]$.