Course outline

This course aims to help you make the transition from high-school-style mathematics to advanced mathematics. Making this transition is not about handling more complicated formulae but about learning mathematical modes of thought. Advanced mathematics asks for crystal-clear arguments. It is also a creative subject with wide horizons. The mathematics covered in this course may be the most challenging you have seen so far, but it might just be the most satisfying. It will give you a head-start in topics studied systematically by math majors (analysis, linear algebra, geometry).

The course will be divided into three quite different parts:

I. From counting to calculus. We will begin by re-examining familiar number systems, and will go on to see how to solve a problem that baffled mathematicians for centuries: how to make calculus make logical sense.

II. Algebra and geometry of linear maps. In this part we will discuss linear maps and matrices, focusing on examples such as rotation about an axis in 3-dimensional space or reflection in a mirror. This material is applicable to computer graphics, mechanical engineering, quantum physics, chemistry, and to any part of science that uses differential equations.

III. The symmetries of plane patterns. The third part will be about the mathematics of symmetry, especially repeating patterns like those found on wallpaper or on brick walls (as in the figure above). This is visual mathematics. You will learn how to identify a pattern by its symmetries. The course will culminate in a modern, “topological” proof that there are precisely 17 kinds of repeating pattern. Examples of all 17 can be found on the walls of the Alhambra Palace in Spain, built in the 14th Century (see below).
Assessment

- **Homework (55%).** Weekly homework problems (except when an extended assignment is due), including questions of various sorts. Some will be routine problems to help learn the material. Others will give you practice in writing proofs. A few will be tough nuts which I hope you will enjoy solving. Homework will be due in class on Thursdays. Your lowest homework grade is dropped.

- **Extended assignments (45%).** There will be three of these—one for each part of the course—each worth 15%. You will choose from a short list of titles. In each of them, I will ask you to explain a topic going slightly beyond what I covered in lectures (so you will need to use the library). I might ask you to explain a particular theorem and its proof, or an application of the theory. In addition, I may ask you to solve a small number of problems relevant to that material. Your work should be around 10 handwritten pages (or the typed equivalent). I’ll be looking for mathematical accuracy, relevance and clarity.