Optimal and Suboptimal Routing Based on Partial CSI in MIMO Ad-Hoc Networks

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Optimal and Suboptimal Routing

Outline

- Introduction & System Model.
- Optimal routing in random ad-hoc networks.
- Single antenna routing.
- MIMO routing.
Statistical optimal routing

Routing in Wireless Ad Hoc Network

Optimal routing
Statistical optimal routing

Routing in Wireless Ad Hoc Network

Optimal routing

Geographically based
Statistical optimal routing

Routing in Wireless Ad Hoc Network

- Optimal routing
- Geographically based
- Using only local knowledge
System model

- Slotted ALOHA MAC ($p_{tx}$).
- PPP distributed nodes ($\lambda$).
- Single / Multiple antennas.
- Local knowledge on nodes in routing zone.
Ergodic Rate Density (ERD)

$$R(\lambda) = \lambda p_{tx} \cdot \mathbb{E} \{\log_2 (1 + \text{SIR})\}$$

- Achievable upper bound on WANETs performance.
- Convenient for analysis.
- Good bounds.
- High complexity, large delay.

George et al, 2013 + 2015

**Graph**

- Novel Upper Bound
- ERD
- Lower bound

**Legend**

- $N=1$ (green)
- $N=3$ (cyan)
- $N=9$ (red)

**Axes**

- Rate Density
- $\lambda$
Asymptotic density of rate and progress (ADORP)

\[ \bar{D}(f(\cdot)) \triangleq \lambda p_{tx} \mathbb{E} \left\{ r_f(M) \log_2 (1 + \text{SIR}_f(M)) \right\} \]

- WANETs performance is measured by Rate × Progress.
- \( f(\cdot) \) is the routing function.
- Use opportunistic relaying.
- No delay constraints.
- Implicit mobility.
Optimal routing

- In the single antenna case

\[ \bar{D}(f(\cdot)) = \lambda p_{tx} E \left\{ r_f(M) \log_2 \left( 1 + \frac{S_f(M)}{J_f(M)} \right) \right\} \]

- Using the Law of Total Expectation

\[ \bar{D}(f(\cdot)) = \lambda p_{tx} E_M \left\{ E_{J|M} \left\{ r_f(M) \log_2 \left( 1 + \frac{S_f(M)}{J_f(M)} \right) \right\} | M \right\} \]

- SO: Statistical Optimal routing function

\[ f_{SO}(M) = \arg\max_{i \in N} r_i \cdot E \left\{ \log_2 \left( 1 + \frac{S_i}{J_i} \right) | M \right\} \]

Depends on the distribution of \( J_i|M \).
Evaluation of the SO metric

- Uses only local knowledge.
- Takes into account interference statistics.
- Can be evaluated using Monte Carlo Simulations.
- Different statistics of nodes inside/outside the routing zone.
BO routing

- Lower bound on ERD

\[ \mathbb{E}\left\{ \log_2 \left( 1 + \frac{S_i}{J_i} \right) \mid \mathcal{M} \right\} \geq p_Z(i, \mathcal{M}) \log_2 \left( 1 + \frac{S_i}{\mathbb{E}\{J_i \mid r > r_Z, \mathcal{M}\}} \right) \]

- BO routing function

\[ f_{BO}(\mathcal{M}) = \arg\max_{i \in \mathcal{N}} p_Z(i, \mathcal{M}) r_i \log_2 \left( 1 + \frac{S_i}{J'_1 + J'_2 + J'_3} \right) \]
NSO routing

- **Narrow** knowledge

\[ M^i = \{ r_i, h_i \} \]

- Narrow Statistically Optimal routing function

\[ f_{\text{NSO}}(M) = \arg\max_{i \in \mathcal{N}} r_i \cdot \mathbb{E} \left\{ \log_2 \left( 1 + \frac{S_i}{J_i} \right) \mid M^i \right\} \]

- Without knowledge on neighbors, the distribution of \( J_i \) is identical for all nodes.

- Interference distribution can be measured locally.

- Complexity is still quite high.
NBO routing

- Lower bound on ERD

\[ p_Z(i, M) \log_2 \left( 1 + \frac{S_i}{\mathbb{E}\{J_i | r_{\min,i} > r_Z, M^i\}} \right) \]

\[ = \text{const} \cdot \log_2 (1 + \gamma S_i) \]

- NBO routing function

\[ f_{NBO}(M) = \arg \max_{i \in N} r_i \log_2 (1 + \gamma \cdot S_i) \]

where

\[ \gamma = \frac{\alpha}{2} \left( \frac{\alpha - 2}{\alpha \pi \lambda p_{tx}} \right)^{\frac{\alpha}{2}} \]

- Considers network parameters.

- Low complexity!
Numerical Results

- Previously published geographic routing schemes
  - **NiC**: Nearest in a cone
    \[ f_{\text{Nearest}}(\mathcal{M}) = \arg\min_i r_i \]
  - **MPR**: Most progress within radius
    \[ f_{\text{MPR}}(\mathcal{M}) = \arg\max_{i:|r_{i,j}| \leq r_{\text{max}}} r_i \]

Need to optimize the routing parameter (MPR)!
ADORP vs $p_{tx}$

Single antenna, $\alpha = 3$
ADORP vs $p_{tx}$

Single antenna, $\alpha = 4$

- **ADORP vs $p_{tx}$**

  - **SO**
  - **BO**
  - **NSO**
  - **NBO**
  - **Nearest**
  - **MPR**

<table>
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<th>$p_{tx}$ (ALOHA transmission probability)</th>
<th>ADORP [bps/Hz/Km]</th>
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Optimal MIMO routing needs to select
- Destination.
- Number of streams.
- Precoding vectors.

Challenges:
- Need to take into account the processing at the receiver.
- The number of streams affects the distribution of the interference.

Simplifying assumptions:
- Tx: eigenbeamforming (equal power streams).
- Rx: Partial ZF (of $N_{ZF}$ nearest transmitters).
- Consider only narrow knowledge.
Multiple antennas WANET

- ADORP

\[ \bar{D}_o \left( f(\cdot) \right) = \lambda p_{tx} \mathbb{E}_M \left\{ B \left( f(M), M, K(M) \right) \right\} \]

Define

\[ B(i, M, K) \triangleq \mathbb{E} \left\{ r_i \sum_{k=1}^{K} \log_2 \left( 1 + \frac{1}{K} \frac{r_i^{-\alpha} W_{i,k}}{J_{i,k}} \right) \middle| M^i \right\}. \]

- \( W_{i,k} \): Effective channel gain at the \( k \)-th stream
- **Partially** known at the transmitter.
NSO routing

Using narrow knowledge

\[ f_{\text{NSO}}(M), K(M) = \arg\max_{i, K} r_i \sum_{k=1}^{K} \mathbb{E} \left\{ \log_2 \left( 1 + \frac{1}{K} \frac{r_i^{-\alpha} \gamma_{i,k}^2 Y_{i,k}}{J_{i,k}} \right) \right\} | M^i \]  

- The distribution of \( J \) depends on the distribution of \( K(M) \).
- High complexity.
- Intractable.
Alternative routing

The optimal routing is given by

\[ f_{\text{FSO}}(M, K) = \arg\max_i r_i \sum_{k=1}^K \mathbb{E} \left\{ \log_2 \left( 1 + \frac{1}{K} r_i^{-\alpha} \gamma_{i,k}^2 \frac{Y_{i,k}}{J_{i,k}} \right) \right\} | M^i \]
Taking the expectation of the numerator and the denominator

\[
f_{LC}(\mathcal{M}), K(\mathcal{M}) = \arg\max_{i,K} r_i \cdot \sum_{k=1}^{K} \log_2 \left( 1 + \frac{1}{K} r_i^{-\alpha} \gamma_{i,k}^2 \bar{Y} \right)
\]

where

\[
\bar{Y} \triangleq \frac{N_R + 1 - K - T_{ZF}}{N_R + 1 - K}
\]

\(\gamma_{i,k}^2\) is the \(k\)-th singular value, and \(T_{ZF}\) is the average number of zeroed streams, and

\[
C_{\alpha,N_{ZF}} \triangleq \frac{2(\lambda p_{tx} \pi)^{\frac{\alpha}{2}} (N_{ZF} - \frac{\alpha}{4})^{1-\frac{\alpha}{2}} \mathbb{E}\{W\}}{\alpha - 2}
\]
ADORP vs $p_{tx}$

Multiple antenna, $N_{ZF} = 1, N_R = 10, \alpha = 3$

![Graph showing ADORP vs $p_{tx}$ for different antenna configurations.](image)
Multiple antenna, $N_{ZF} = 2, N_R = 10, \alpha = 3$
Multiple antenna, $N_{ZF} = 3$, $N_R = 10$, $\alpha = 3$
Conclusions

- Presented novel routing metrics.
- Based on optimization of the ADORP bound.
- Simple to evaluate locally.
- Uses only local knowledge and exploits the statistics of the interference.
- Close to optimal, outperforms traditional schemes.
Thank you!
**ERD Lower Bound**

- **Rewrite**

\[
\bar{D}(f(\cdot)) = \lambda p \mathbb{E}_M \left\{ G(f(M), M) \right\}
\]

where

\[
G(i, M) = \mathbb{E}_{J|M} \left\{ r_{i,0} \log_2 \left( 1 + \frac{\rho S_i}{J_i} \right) \right\}.
\]

- **Routing Zone and Threshold Zone**
**ERD Lower Bound**

- $p_Z(i, M)$: probability that no transmitter within distance $r_Z$ from node $i$:

$$
p_Z(i, M) = \begin{cases} 
(1 - p)^{N_Z,i}, & \text{if } \|r_i\| + r_Z < r_R \\
 e^{-\lambda p B_T,i} (1 - p)^{N_Z,i}, & \text{o.w.}
\end{cases}
$$
ERD Lower Bound

- Lower bound

\[ G(i, M) = (1 - p_Z(i, M)) \mathbb{E}\left\{ r_i \log_2 \left( 1 + \frac{\rho \cdot S_i}{J_i} \right) \middle| r_{\text{min}, i} \leq r_Z, M \right\} \]

\[ + p_Z(i, M) \mathbb{E}\left\{ r_i \log_2 \left( 1 + \frac{\rho \cdot S_i}{J_i} \right) \middle| r_{\text{min}, i} > r_Z, M \right\} \]

\[ \geq p_Z(i, M) \mathbb{E}\left\{ r_i \log_2 \left( 1 + \frac{\rho \cdot S_i}{J_i} \right) \middle| r_{\text{min}, i} > r_Z, M \right\} \]

\[ = p_Z(i, M) \cdot r_i \log_2 \left( 1 + \frac{\rho \cdot S_i}{\mathbb{E}\{ J_i \mid r_{\text{min}, i} > r_Z, M \}} \right) \]

- Eventually,

\[ \mathbb{E}\{ J_i \mid r_{\text{min}, i} > r_Z, M \} = \bar{J}_1^i + \bar{J}_2^i + \bar{J}_3^i \]
Partial Zero Forcing (PZF)

- Cancels its $N_{ZF}$ nearest transmitters.
- $T_{ZF}$: # of inter-streams to be canceled.
- Remaining degrees of freedom (DOF)

$$L = N_R - T_{ZF} - (K_0 - 1).$$

- Set of $N_{ZF}$ indices of undesired transmitters

$$\mathcal{N}_{ZF}^{\text{Inter}} = \{j \in \Phi_T : r_j \leq r_{ZF}\}.$$

- Set of its $(K - 1)$ intra-streams

$$\mathcal{N}_{ZF,k}^{\text{Intra}} = \{1, 2, ..., k - 1, k + 1, ..., K\}.$$