Preliminary Examination in Topology: August 2015

Algebraic Topology portion

Instructions: Do all three questions.

Time Limit: 90 minutes.

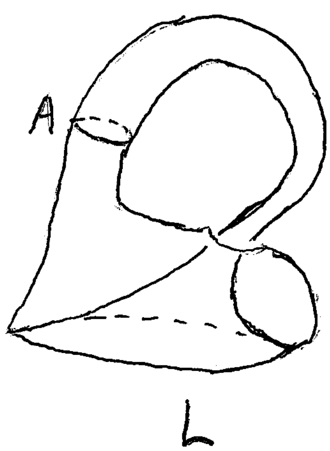
1. Consider the two 2-complexes K and L drawn below. K is a punctured double torus with boundary curve J with a disk added along a meridian curve. L is a Klein bottle with a meridian curve A drawn on it. Let X be the complex obtained from $K \bigcup L$ by identifying J with A via a piecewise linear homeomorphism.

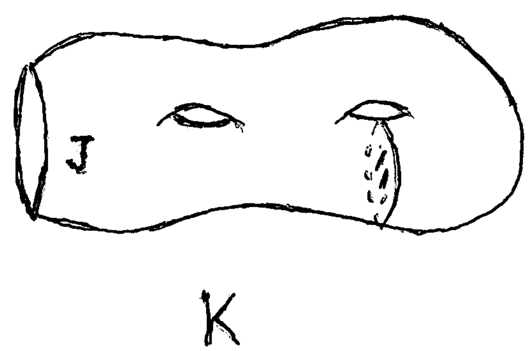
a) What are the fundamental groups of K and L? Briefly justify your answers.

b) Using the decomposition of X as $K \bigcup L$, compute the fundamental group of the complex X using Van Kampen's Theorem.

c) What are the homology groups of K and L? Briefly justify your answers.

d) Using the decomposition of X as $K \bigcup L$, compute all the homology groups of the complex X using the Mayer-Vietoris Theorem.





2. Describe all spaces that can be 4-fold covering spaces of the connected sum of five projective planes. Describe covering maps in each case with a picture. Explain why you know your list of spaces is complete.

3.

a) Using insights from algebraic topology, prove that the free group on two elements has a normal subgroup of index three.

b) Using insights from algebraic topology, prove that the free group on two elements has a subgroup of index three that is not normal.

c) Are the two subgroups above necessarily free groups? If so, what are their ranks? Why?