

Algebra Prelim Part B January 2016

Do two of the three questions below. Please indicate which questions you want graded.

1. Consider $f(x) = x^4 - 2x^2 - 2 \in \mathbf{Q}[x]$. Observe that $f(x) = 0$ has 4 distinct roots: $\pm\alpha, \pm\beta$ where $\alpha \in \mathbf{R}$ and $\beta \in \mathbf{C} \setminus \mathbf{R}$.

- a) Determine the degree of the splitting field E of $f(x)$ over \mathbf{Q} .
- b) Prove that $\text{Gal}(E/\mathbf{Q})$ is isomorphic to the dihedral group of order 8.
- d) Determine a primitive generator of E/\mathbf{Q} .
- c) Determine all the subfields of E and identify the ones that are Galois over \mathbf{Q} .

2. Show that the polynomial $f(x) = x^4 + x + t \in \mathbf{F}_2(t)[x]$ is irreducible and compute the Galois group of the splitting field of $f(x)$ over $\mathbf{F}_2(t)$

3. Find a polynomial of degree 5 in $\mathbf{Q}[x]$ (polynomials over the rational numbers) whose splitting field has Galois group isomorphic to $\mathbf{Z}/5\mathbf{Z}$ (the cyclic group of order five).