Algebraic Topology Prelim Exam August 15, 2023, 12:30–14:30

Solve all three problems.

Problem 1. Let X be an orientable surface of genus 2, and Y be the connected sum of four copies of $\mathbb{R}P^2$. Find a common covering space, together with explicit covering maps from it to X and Y.

Problem 2. $\mathbb{Z}/2$ is the only nontrivial group that can act freely on $S^n,$ for n even.

Problem 3. $S^a \times \mathbb{R}P^b$ and $S^b \times \mathbb{R}P^a$ are homeomorphic if and only if a = b.