

# UT Math Club Problem of the Month

October, 2011

**Problem** (*Wall Street Fez Hat Fiasco*) Demonstrators recently assembled in Wall Street to protest (among other things) the government bail outs which occurred as a response to the late-2000s financial crisis. In order to confuse and break the moral of the protesters, the local police arranged the following scenario. First, the protesters were lined up single-file. Next, a police officer went down the line, back to front, and put a colored fez hat on the head of each protester. Finally, the protesters were each asked to guess the color of the hat on their own head, starting at the back of the line and moving forward one at a time. Each protester who guessed correctly would be released, and each protester who guessed incorrectly would be sprayed with mace, shocked with a taser, and then arrested. Luckily for the protesters, they'd heard of just such a tactic being employed elsewhere, and were prepared with a strategy which ensured that all but one of the protesters would be released.

1. Suppose that there are  $n \geq 1$  protesters and  $k \geq 2$  colors for the hats. Also assume that the protesters know the value of  $k$  in advance, and can see the colors of the hats in front of them, but not the colors of the hats behind them or the color of their own hat. Also assume that the only means of communication available to the protesters is their guess; they cannot make any other sound or take any other action. What was the strategy of the protesters?
2. Now suppose that there are countably infinitely many protesters (but still finitely many colors for hats). Show that there is still a strategy which ensures that all but finitely many of the protesters are released. You can assume that the protesters are infinitely intelligent and can see infinitely far down the line, and also I guess that the Wall Street district can accommodate an infinitely long line of people. So you may have to suspend your disbelief a little bit.
3. Is there an optimal strategy in the case of infinitely many protesters? In other words, is there a way to minimize the number of protesters who are arrested?

**Submission Deadline:** Monday, October 31<sup>st</sup> by 11:59 PM