

# UT Math Club Problem of the Month

September 9, 2009

**Problem I** (*An Interesting Inequality*) Let  $S \subset \mathbb{R}^3$  be a finite collection of points. Define the projection maps  $\pi_1, \pi_2, \pi_3 : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  by  $\pi_1(x, y, z) = (y, z)$ ,  $\pi_2(x, y, z) = (x, z)$  and  $\pi_3(x, y, z) = (x, y)$  and let  $S_1 = \pi_1(S)$ ,  $S_2 = \pi_2(S)$  and  $S_3 = \pi_3(S)$ . Prove the following inequality:

$$|S| \leq \sqrt{|S_1|} \sqrt{|S_2|} \sqrt{|S_3|}$$

where  $|S|, |S_1|, |S_2|$  and  $|S_3|$  denotes the number of elements in the sets  $S, S_1, S_2$  and  $S_3$ , respectively.

**Problem II** (*A Seating Arrangement*) The A&M math club is going out for pizza at the Round Table Pizzeria. There are  $2n$  members of their math club and coincidentally the Round Table Pizzeria has a single round table with exactly  $2n$  seats. Unfortunately, due to the ornery nature of A&M students, each member has exactly  $n - 1$  enemies (within the math club). Prove that there exists a seating arrangement so that every member of the A&M math club can be seated at the table without having to sit next to one of their enemies.

**Submission Deadline:** Wednesday, September 30 by 11:59 PM