

# UT Math Club Problem of the Month

January 26, 2010

**Problem I** (*Polynomial Minimum*) Consider the following proposition:

**Proposition** If  $p \in \mathbb{R}[x, y]$  is a polynomial in the variables  $x$  and  $y$  with real coefficients then there exists a point  $(a_0, b_0) \in \mathbb{R}^2$  such that  $|p(a_0, b_0)| \leq |p(a, b)|$  for all  $(a, b) \in \mathbb{R}^2$ , i.e. the absolute value of  $p$  achieves a global minimum.

Is the proposition stated above true or false? If it's true, provide a proof. If it's false, produce a counter-example; i.e. find a polynomial which does not achieve a global minimum.

**Problem II** (*The Picture-Frame Problem*) American media tycoon Ted Turner puts two nails into the wall over his fireplace and hangs a picture frame (possibly one containing a picture) from these two nails by a length of string. Each end of the string is attached to one corner of the picture frame, and is wrapped around the two nails in such a way that the picture frame is held up by the nails, but will fall to the ground if either one of the two nails is removed. Meanwhile, on the other side of the globe, Australian-born global media mogul and infamous one-upmanship artist Rupert Murdoch puts three nails into the wall over his fireplace and hangs a picture frame (possibly one containing a picture) from these three nails by a length of string. Each end of the string is attached to one corner of the picture frame, and is wrapped around the three nails in such a way that the picture frame is held up by the nails, but will fall to the ground if any one of the three nails is removed. Not to be outdone, Donald Trump puts three nails into the wall over his fireplace and hangs an empty picture frame from these three nails by a length of string. Each end of the string is attached to one corner of the picture frame, and is wrapped around the three nails in such a way that the picture frame is held up by the nails, but will fall to the ground if any two of the three nails is removed.

How did each of the three men manage their amazing feats of picture frame hanging?

**Submission Deadline:** Saturday, April 30<sup>th</sup> by 11:59 PM