Visual Image of Cooper Pairing in Superconductors

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Abstract

In this paper, we provide a dynamic geometric imaging of the isosymmetries, which characterize the mutual contact/penetration of paired electron wavepackets in the isostandard model of cooper pairs in superconductors. This is accomplished by modifying the theoretical framework developed by researchers at Howard University. In their model, the Howard group based their approach on the classical Hamiltonian $H_i = \sqrt{(\mathbf{p}_i - e\mathbf{A}/c)^2 + (m_i c^2 + V)^2}$, $i = 1,2$, which showed the clear power of the isotopic method. As an application, in this paper, we consider a representation of the usual quantum mechanical non-relativistic quasiparticle energy in (**k**-space) of a Cooper pair, $\left(E_{\mathbf{k}} = \sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}}\right)$, as an eigenvalue of a 4×4 Hamiltonian matrix, $H_{\mathbf{k}} = \varepsilon_{\mathbf{k}} g \pm \Delta_{\mathbf{k}} \hat{g}$, involving the conventional Minkowski metric $g = diag[1,-1,-1,-1]$ and its isotope $\hat{g} = \beta g$. The β-matrix here is the 4×4 antisymmetric matrix introduced by Dirac in his construction of a positive energy relativistic wave equation for an integral spin particle. We find a new class of dynamic geometric images in space-time as isominkowskian spaces define by, $s^2 = x^\mu \left(g_{\mu\nu} \pm \xi \hat{g}_{\mu\nu}\right) x^\nu$, $0 \le \xi \le 1$, which represents a pair of spheres in contact when $x^0 = s$ and $\xi = 1$. The physical implications are discussed.

Introduction

It has been a recurrent dream in physics to unify quantum theory and the metric of the underlying space-time geometry as a prelude for incorporating internal nonlocal effects in composite systems as well as Einstein's general relativity theory (of gravitation) into quantum mechanics. While the traditional paradigm for achieving this has been to prescribe the space-time metric a priori and construct conventional quantum theory within it, other alternative paradigms exist and are worth examining for insight. In particular, one could equally well reverse the traditional paradigm by first prescribing a sufficiently general quantum mechanical equation of motion and then determining the space-time metric compatible with it a posteriori via both mathematical and physical self-consistency. The pursuit of this latter paradigm led Santilli in 1978 to propose a generalization of the underlying product of two operators. If A, B are two operators in traditional quantum mechanics, so that $[A,B] = AB - BA$ defines a Lie-product, then the product AB is replaced by $A \bullet B$, where $A \bullet B \triangleq ATB$ for some fixed operator T, such that

$$[A,B]_T = A \bullet B - B \bullet A , \qquad (1)$$

is also a Lie-product, now called the Lie-Santilli product. This leads to a generalized form of Heisenberg's equation of motion and space-time geometry in the following self-consistent way:

$$i\hbar \frac{dA}{dt} = ATH - HTA, \quad x^{\mu} \hat{g}_{\mu\nu} x^{\nu} = s^2 , \; (\hat{g} = T^{-1}). \qquad (2)$$

Space-time symmetries related to conventional symmetries by such Lie-isotopic lifting (transformations) are called isosymmetries; and the resulting generalization of conventional quantum mechanics is called "hadronic mechanics" [1].

The most salient feature of the Lie-isotopic lifting transformation in hadronic mechanics is that it preserves the established axioms of quantum mechanics under generalized interactions built into the isotopies characterized by a generalized Planck constant unit of quantum mechanics ($\hat{\hbar} = \hbar \hat{g}$). For this reason, one of the most intriguing problems of hadronic mechanics has been the geometrical realization of the nonlocal effects associated with the generalized Planck unit $\hat{\hbar}$.

Recently, some progress has been made in this respect by researchers at Howard University [2], [3]. The group achieved explicit non-trivial construction of isosymmetries in three-dimensional space, by using computer visualization techniques in a dynamical context. This was done by postulating a classical relativistic Hamiltonian for each particle of a two particle Hamiltonian system of the form:

$$H_i = \sqrt{\left(\mathbf{p}_i - e\mathbf{A}/c\right)^2 + \left(m_i c^2 + V\right)^2} , \; i = 1,2 \qquad (3a)$$

where the potential energy function,

$$V(t) = -mc^2 \left\{ 1 - \sqrt{a(t)x^2 + b(t)(y^2 + z^2)} \right\} , \qquad (3b)$$

is heuristically associated with an isotopic element defined by:

$$T = T(t) = \left[\sqrt{a(t)x^2 + b(t)(y^2 + z^2)} \right]^{-1}, \quad \left(a(t) = 1 + 3t,\ b(t) = 1 - t \right) \tag{4}$$

which characterizes an isotopic lifting (transformation) of the unit sphere in 3-dimensional space in the form:

$$\begin{aligned} 1 = x^2 + y^2 + z^2 \equiv x^i g_{ij} x^j \to x^i \hat{g}_{ij} x^j &= x^i T g_{ij} x^j \\ &\equiv \frac{x^2 + y^2 + z^2}{\sqrt{a(t)x^2 + b(t)(y^2 + z^2)}} = 1. \end{aligned} \tag{5}$$

This equation leads to a representation of the isosymmetries of a sphere as a composite pair of quadratic surfaces defined by:

$$\begin{aligned} & a(t)x^2 + b(t)(y^2 + z^2) = \left(x^2 + y^2 + z^2 \right)^2 \Rightarrow \\ & \left(x^2 + y^2 + z^2 \right) + t\left(3x^2 - y^2 - z^2 \right) = \left(x^2 + y^2 + z^2 \right)^2. \end{aligned} \tag{6}$$

As a function of t, this pair evolves continuously as shown in Figure 1 for $t \in [0,1]$, from a pair of unit and point spheres (Fig. 1a):

$$t = 0, \ \Rightarrow \left(x^2 + y^2 + z^2 \right)\left(x^2 + y^2 + z^2 - 1 \right) = 0, \tag{7}$$

to a pair of unit spheres in contact (Fig. 1d):

$$t = 1, \ \Rightarrow \left[(x-1)^2 + y^2 + z^2 - 1 \right]\left[(x+1)^2 + y^2 + z^2 - 1 \right] = 0. \tag{8}$$

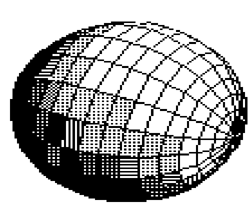

Figure 1a

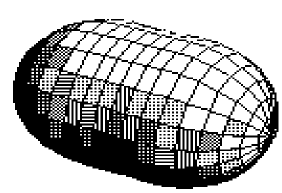

Figure 1b

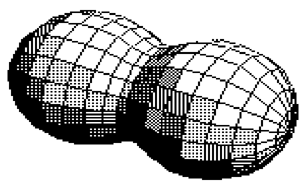

Figure 1c

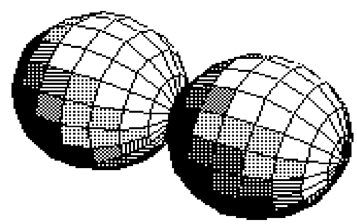

Figure 1d

Without question, the above visualization technique has provided a new profile of the isosymmetry as a geometrical process characterizing not only the deformation of isolated geometric objects (two spheres in the above case) but also their mutual contact/penetration or fusion into a single composite object, as envisaged in the composite model of the strongly interacting particles (hadrons). Consequently, the technique leads to at least two open problems. The first problem is to identify the metric of the isosymmetric space of the composite object, which is characterized by a system of quadratic surfaces, such as equation (6b). The second problem is to provide a framework, which will allow us to understand the implications of the geometrical contact/penetration of the constituent particles of a composite system on the intrinsic characteristics of the bound particles. These problems are investigated in this paper for the case of two electrons represented by their spherical s-wavepackets forming a Cooper pair in isosuperconductivity theory ([4], [5], [6]).

Quantum Represention

Our investigation is made possible by the fact the (square root) mathematical structure of the classical relativistic Hamiltonian model in equation (4) is also realized in the non-relativistic quantum mechanics as a Cooper pair in a superconductor in the form of the usual quasiparticle energy of a pair given in **k**-space by the expression:

$$E_{\mathbf{k}} = \sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}\,, \tag{9}$$

where $\varepsilon_{\mathbf{k}}$ is the kinetic energy (measure relative to the Fermi level) and $\Delta_{\mathbf{k}}$ is the energy gap. However, instead of introducing the potential V heuristically, we proceed, as in reference [4], by representing (9) by a Hamiltonian 2×2 matrix:

$$H_{\mathbf{k}} = \begin{bmatrix} \varepsilon_{\mathbf{k}} & \pm\Delta_{\mathbf{k}} \\ \pm\Delta_{\mathbf{k}} & \varepsilon_{\mathbf{k}} \end{bmatrix} = \varepsilon_{\mathbf{k}}\tau_3 \mp \Delta_{\mathbf{k}}\hat{\tau}_3 \tag{10}$$

whose eigenvalue is $E_{\mathbf{k}}$, where

$$\tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\ \hat{\tau}_3 = \beta\tau_3 \equiv \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix},\ \beta = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \tag{11}$$

This lends itself to a further generalization, as suggested in reference [4], in the form of the following 4×4 Hamiltonian matrix:

$$H_{\mathbf{k}} = \varepsilon_{\mathbf{k}} g \pm \Delta_{\mathbf{k}} \hat{g}\,, \tag{12}$$

where

$$g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix},\ \hat{g} = \beta \mathrm{g} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

are the usual Minkowski space metric and its isotopy given by

$$\alpha_\mu \beta \alpha_\nu + \alpha_\nu \beta \alpha_\mu = 2\beta g_{\mu\nu} \tag{13}$$

which involves the following matrices: $\alpha_0 = I$,

$$\alpha_1 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\ \alpha_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$\alpha_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix},\ \beta = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}.$$

These are the matrices used by Dirac to construct his positive-energy relativistic wave equation for an integral spin (composite) particle. Explicitly, from equation (12),

$$H_{\mathbf{k}} - E = \begin{bmatrix} \varepsilon_{\mathbf{k}} - E & 0 & -\Delta_{\mathbf{k}} & 0 \\ 0 & -\varepsilon_{\mathbf{k}} - E & 0 & -\Delta_{\mathbf{k}} \\ -\Delta_{\mathbf{k}} & 0 & -\varepsilon_{\mathbf{k}} - E & 0 \\ 0 & \Delta_{\mathbf{k}} & 0 & -\varepsilon_{\mathbf{k}} - E \end{bmatrix} \tag{14}$$

and, therefore, equation (9) is recovered in the form:

$$\det\|H_{\mathbf{k}} - E\| = \left(E^2 - \varepsilon_{\mathbf{k}}^2 - \Delta_{\mathbf{k}}^2\right)\left[\left(E + \varepsilon_{\mathbf{k}}\right)^2 + \Delta_{\mathbf{k}}^2\right] = 0\,, \tag{15}$$

which shows that equation (1) leads indeed to a generalization of equation (4).

Now, we observe that, with $(x^{\mu}) = (ct, x, y, z)$, the isominkowskian space characterized by the quadratic form:

$$x^{\mu}\left(g_{\mu\nu} \pm \xi\beta g_{\mu\nu}\right)x^{\nu} \equiv \left[(ct)^2 - x^2 - y^2 - z^2\right] \pm \xi cty = s^2,\ 0 \le \xi \le 1 \tag{16}$$

is equivalent, when $ct = s,\ \xi = 1$, to a pair of spheres in contact along the y-axis , i.e., equation (15) can be rewritten in the form of equation (8). This is the result we are after.

We observe that, with our approach it is the two-dimensional nature of the isometric space with $\hat{g}_{\mu\nu} = \beta g_{\mu\nu}$ that permits the mutation of the total angular momentum of the Cooper pair to the value zero. We conclude therefore that equation (15) provides a visual image of the contact-overlap interactions between the paired electrons in the Cooper pair which is consistent with isosuperconductivity theory ([4]-[6]).

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