Transient Nature of Generalized Coulomb Gauge – A Mathematical Key to Color Confinement and Mass-Gap Problems of Pure Yang – Mills Field Theory

Rajan Dogra

Department of Technical Education, U.T., Chandigarh,
3291, Sector 27 –D, Chandigarh – 160 019, India,
E-mail: rajandogra_2000@yahoo.com

Abstract

In order to avoid non-locality of the action when generalized Coulomb gauge is imposed, the implementation of the non-abelian Gauss law for infinitesimal time-period over the space of gauge potentials in path-integral method of pure Yang – Mills theory provides deep insight into the **color confinement** and **mass-gap** problems in the pure Yang – Mills fields i.e the question of glueball spectrum.

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1. INTRODUCTION:

The brief quantization history of the pure Yang – Mills theory as outlined in [1] is referred to in this section. If one looks at this history of designing a quantum theory for the non-abelian gauge fields, one comes across numerous methods for the same. To begin with, a Hamiltonian formulation, derived from a covariantly gauge fixed Lagrangian á la QED, was used by Yang – Mills in their original article [2] for the non-abelian gauge fields. For this, they have quantized all the four components of the gauge potential. But they did not bother to find the physical states that were, earlier, found to be non-trivial in the covariant (or Lorentz) gauge by Gupta-Bleuler formulation of QED [3].

In view of this non-trivial nature, Schwinger proposed to use Coulomb gauge instead for the Hamiltonian formulation [4]. But the Hamiltonian, which

Schwinger obtained after incorporating relativistic invariance, had non-standard form for the kinetic energy. As such, this Hamiltonian of Schwinger had no practical utility. This forced Feynman, in 1976, to state that "Schwinger, after a lot of hard work found the Hamiltonian…., but…progress in this direction ceased" [5]. In the intervening time, Hamiltonian formulation had lost its sheen as a method for designing a quantum theory for the non-abelian gauge fields as people showed interest in an alternative, path-integral method.

This was primarily due to path-breaking research work undertaken by Faddeev and Popov [6] who showed the appearance of the ghost particles by insertion of their famous determinant in the path-integral measure. In-fact, these ghost particles established beyond doubt the unitary nature of the Feynman amplitudes in the earlier work of Feynman [7] and De Witt [8]. Following this, the path-integral method, as ushered by Faddeev and Popov work [6], soon became the method of quantizing non-abelian gauge fields and an elegant way to understand Feynman Rules and perturbation theory.

However, in the light of Gribov work [9] the inadequacy of the Faddeev – Popov determinant method beyond perturbation theory became evident. This inadequacy leads to the fact that the presence of Gribov copies is absolutely necessary and that the multiple representations of the gauge orbits is a must. In this paper, it is shown that the canonical momenta conjugate to time-component A_0 of the Yang-Mills gauge potential does not exist due to the gauge invariance of Lagrangian L in Eq. (2) of this paper. Neglecting this problem, the mathematically constructed Hamiltonian H in Eq. (4) of this paper is not unique as the dynamical equations can be modified by both time-dependent and time-independent gauge transformation.

The time-dependent gauge freedom of H manifests itself through the presence of arbitrary term A_0 in Eq. (4) of this paper as none of the Yang–Mills equations involves the time-derivative of A_0. The time-independent gauge freedom of H manifests itself in the absence of Gauss law in the dynamical part of the Yang –Mills equations reproduced by H, through the Hamiltonian equations of motion. Both aforesaid time-dependent and time-independent gauge freedoms of H are simultaneously addressed by substituting $A_0 = A_0\{ A_k, \partial_0 A_k\}$in L of Eq. (2) of this paper. This substitution leads to impossibility of expressing generalized velocity $\acute{A}^k$ in terms of generalized co-ordinate A_k and conjugate momenta π^k in Eq. (10a) of this paper.

At this stage, the generalized coulomb gauge$\nabla_k(A)\ \acute{A}^k = 0$ allows construction of H from Eq. (4) in usual manner by using $A_0 = A_0\{ A_k\}$in Eq. (10a) of this paper. But at the same time this generalized coulomb gauge$\nabla_k(A)\ \acute{A}^k = 0$ makes the generalized velocities $\acute{A}^k$ the dependent quantities that cannot be used in the construction of canonical momentas in Eq. (3) of this paper. This problem of dependent generalized velocities $\acute{A}^k$ is resolved when light is thrown on the transient nature of gauge transform by which generalized coulomb gauge$\nabla_k(A)\ \acute{A}^k = 0$ is realized. In other words, the gauge fixing by generalized coulomb gauge for infinitesimal time-period only does allow the generalized velocities $\acute{A}^k$ to be treated as the independent quantities in mathematical sense.

Consequent upon the same, the construction of H proceeds in usual manner from short-lived canonical momentas. This newly constructed short-lived

Hamiltonian H has no time-dependent gauge freedom, but definitely has time-independent gauge freedom due to improper gauge fixing (by generalized coulomb gauge) also called Gribov ambiguity. In order to avoid non-locality of the action, this Gribov ambiguity leads to multiple representation of a gauge orbit by its gauge copies. Consequently, these Gribov copies are logically nothing but physical states that are short-lived in nature. Only this concept of short-lived Gribov copies as physical states provides deep insights into confinement and mass-gap generation problems in this paper.

2. CANONICAL APPROACH

For the initial formulation of the classical Yang – Mills equations, I would refer to [10] in this and subsequent para. Classically, the gauge theory is illustrated by the theory of electromagnetism for which the aeblian group $U(1)$ is the gauge group. If we locally view the $U(1)$ gauge connection, as denoted by A, as one-form on space-time, then the two form $F = dA$ is nothing but curvature or electro-magnetic field tensor in space-time. In this terminology, the Maxwell's equations are represented by $0 = dF = d{*F}$, where * is the Hodge duality operator [10].

Classically, by substituting the abelian group $U(1)$ with a more general compact gauge group $G = SU(3)$, the curvature of the space-time is changed to $F = dA + A \wedge A$, and Maxwell's equations are transformed to the Yang-Mills equations, $0 = d_A F = d_A {*F}$, where d_A denotes the gauge-covariant extension of the exterior derivative. These Yang-Mills equations can be validated by deriving them from the pure Yang-Mills Lagrangian [10]

$$L' = (1/4g^2) \int \mathrm{T_r}\, F \wedge {*F} \qquad (1)$$

where $\mathrm{T_r}$ denotes an invariant quadratic form on the Lie algebra of G.

In the relativistic Lagrangian of Equation (1), the time and space indices have been treated on equal footing. If the time index, as a parameter, is treated on unequal footing with space indices in the Lagrangian of Equation (1), we get new non-relativistic Lagrangian L of space indices as

$$L' = \int_{t_1}^{t_2} dt\, L \qquad (2)$$

where integration is from time instant t_1 to t_2.

The very first step towards canonical quantization involves the conversion of the classical Lagrangian L of Equation (2) into a Hamiltonian one. The standard procedure [11] for this is to define canonical momenta π^{μ} by the following mathematical relations.

$$\pi^{\mu} = \partial L/\partial \acute{A}_{\mu} \qquad (3)$$

where $\mu = 0,1,2,3$ indicates the Minkowski space-time indices and $\acute{A}_{\mu}$ are the generalized velocities.

Now, the canonical momenta π^0 corresponding to the time inde x $\mu = 0$ in the above mathematical formula vanishes and as such, for this non-relativistic Lagrangian L the canonical quantization faces problem [11] at the outset of the quantization process itself in the Dirac formulation for the constrained systems [12, 13]. This fundamental mathematical problem of the vanishing canonical momenta π^0 originates from the gauge invariance of L and indicates non-unique Hamiltonian as the dynamical Yang – Mills equation can always be modified by arbitrary time-dependent gauge transformations.

In view of the above mathematical formulation, the gauge invariance of the Lagrangian L of the Equation (2) must be overcome for arriving at the physical interpretation of the quantization problem of vanishing canonical momenta π^0. This can be done by fixing the gauge with the help of a gauge condition. Let us see how this happens: Neglecting the problem of the vanishing canonical momenta π^0, corresponding to temporal co-ordinate and choosing the time index as a parameter to begin with, if the corresponding pairs of spatial components of Yang – Mills gauge field A_k (where the space indices k = 1,2,3) and canonical momentas π_k (where the space indices k = 1,2,3), corresponding to spatial co-ordinates, are chosen as canonically conjugate variables in the non-relativistic Lagrangian L of the Equation (2), it is possible to mathematically construct [11] a Hamiltonian H through the Legendre transformation given as

$$H = \int \pi_k . \acute{A}^k \, d^3x - L \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (4)}$$

The above H, through the Hamiltonian equations of motion, reproduces [11] the dynamical part of the Yang – Mills equations but Gauss's law is absent in such a system. No physical applications, like working out the physical particle spectrum of QCD from the Hamiltonian H, are possible before Gauss law is properly incorporated into the Hamiltonian formalism [11].

It is shown in [14] that the starting point, for the implementation of the Gauss law in Hamiltonian formalism, is the non-abelian Gauss law that is obtained as a Lagrange equation of motion for $\mu = 0$ when Hamilton's action principle is invoked for the Lagrangian of Equation (1), i.e.,

$\nabla_k(A)\nabla^k(A)A^0 - \nabla_k(A)\, \acute{A}^k = 0$ Where $\nabla_k(A)$ is 'covariant gradient'. _______________ (5)

In-fact, with time index $\mu = 0$ acting as a parameter in the time interval (t_1, t_2) and with space indices (k = 1, 2, 3) acting as independent variables, it is mentioned in [14] that the aforementioned non-abelian Gauss law in Equation (5) is a system of linear, elliptic partial differential equations determining the (matrix valued) potential component A_0 for given space components A_k & their time derivatives $\partial_0 A_k$ and it can be solved by assuming the existence of unique solution A_0 as a functional of A_k and their time derivatives $\partial_0 A_k$, i.e.,

$$A_0 = A_0\{ A_k, \partial_0 A_k\} \qquad (6)$$

At this stage, a natural question, as raised in [14], is then whether the pure Yang – Mills Lagrangian *L'* of the Equation (1) can be used for deriving a canonical structure of the pure Yang – Mills when the potential component A_0, as given by the Equation (6) above, is a solution of the aforementioned non-abelian Gauss law in Equation (5). For answering this question, it is mentioned in [14] that the first step is to use the Lagrangian L of the Equation (2) that has been derived from *L'* to begin with and then substitute the aforesaid unique solution A_0, as given by the Equation (6) above, into this L to get new Lagrangian L_0 as given below,

$$L_0 = (-1/2) \int_V d^3x\, (\nabla_k(A_k)\, A_0\{A_k, \partial_0 A_k\} - \acute{A}_k, \nabla^k(A^k)\, A^0\{A^k, \partial^0 A^k\} - \acute{A}^k)$$
$$- (1/4) \int_V d^3x (G_{kl}(A_k), G^{kl}(A^k)) \qquad (7)$$

where $G_{kl} = \partial_k A_l(x) - \partial_l A_k(x) - ig[A_k(x), A_l(x)]$ and k,l are denoting space indices ranging from 1 to 3.

This new Lagrangian L_0 must reproduce the Lagrange equations of motion for k = 1,2,3 when Hamilton's action principle is invoked. Towards this goal, it is quite obvious to verify the following result beforehand as is done in [14],

$$\delta \int dt\, L_0 = - \int dt \int_V d^3x\, (\delta A_k, \nabla_0(A)\, (\nabla^k(A)\, A^0\{ A^k, \partial^0 A^k\} - \acute{A}^k) - \nabla_l G^{kl}(A))$$
$$- \int dt \int_{\partial V} d^2\sigma_k\, ((\delta A^0\{ A^k, \partial^0 A^k\}, \nabla^k(A)\, A^0\{ A^k, \partial^0 A^k\} - \acute{A}^k) \qquad (8)$$

where time integration is from time instant t_1 to t_2.

Now, it is mentioned in [14] that the boundary conditions applicable to the A_0 are required to be considered at this stage. If the domain V is taken as all R^3 and accordingly, the boundary of the domain V in above Equation (8) is taken at spatial infinity ($R\rightarrow\infty$), then the vanishing of the second surface term in above Equation (8) is taken to be equivalent to the following condition in [14],

$$\lim_{R\rightarrow\infty} \int_{V,\ |x|=R} d\Omega R^2 \, (\delta A^0\{A^{(r)}, \partial^0 A^r\}, \nabla^{(r)}(A)\, A^0\{A^{(r)}, \partial^0 A^r\} - \acute{A}^{(r)}) = 0 \qquad (9)$$

where (r) denotes the radial component of the corresponding quantity.

As such, subject to the above boundary condition in Equation (9) for all admissible variations of $A_{(r)}$ and $\partial_0 A_{(r)}$, we get the following Lagrange equations of motion for k = 1,2,3 for L_0 from Equation (8) in the light of the variational principle $\delta \int dt\, L_0 = 0$, i.e.,

$$\nabla_0(A)\, (\nabla^k(A)\, A^0\{ A^k, \partial^0 A^k\} - \acute{A}^k) - \nabla_l G^{kl}(A)) = 0 \qquad (10)$$

Further, it is mentioned in [14] that the vanishing of the surface term in the Equation (9) above depends upon the assumed asymptotic behavior of the independent variables A_k and their time derivatives $\partial_0 A_k$ as well as the on the boundary conditions (at spatial infinity) of the dependent variable A_0.

As such, given that the surface term in the Equation (9) above does vanish under these asymptotic and boundary conditions being fulfilled and accordingly, the L_0 does reproduce the Lagrange Equations (10) of motion for k = 1,2,3, then the substitution of L_0 of the Equation (7) into the Equation (3) above leads to

$$\pi^k = \partial L_0/\partial \acute{A}^k = (\nabla^k(A)\, A^0\{A^k, \partial^0 A^k\} - \acute{A}^k) \qquad (10a)$$

From above expression for canonical momentum π^k, it is impossible to find $\acute{A}^k$ in terms of π^k and A^k because here A^0 is a functional of A^k and their time derivatives $\acute{A}^k = \partial^0 A^k$. One can convert the aforesaid impossibility into possibility by imposing an attainable gauge fixing at Lagrangian level in Gauss law of Equation (5) i.e., for every Yang – Mills field configuration, there must exist a gauge-transformed Yang – Mills field configuration that satisfies the following generalized coulomb gauge fixing condition, i.e.,

$$\nabla_k(A)\, \acute{A}^k = 0 \qquad (11)$$

With the substitution of Equation (11) in equation (5), the equation (6) is modified as

$$A_0 = A_0\{ A_k\}$$

As such, one can now straightforwardly express the generalized velocity $\acute{A}^k$ in terms of generalized co-ordinate and momenta variables by using $A_0 = A_0\{ A_k\}$ in the Equation (10a). The construction of the Hamiltonian H from Equation (4) should now proceed in usual manner. However, it is mentioned in [14] that with the use of the above generalized coulomb gauge fixing condition of the Equation (11), the generalized velocities $\partial_0 A_k$ are no longer independent quantities and as such, cannot be used for the construction of canonical momentas in the Equation (3). As such, an alternate line of reasoning that leads to a proper canonical formalism is given below.

It is proved in [15] that the above Equation (11) is a bona fide gauge fixing condition, the existence of which depends on an elliptic linear partial differential equation for a Lie-algebra valued quantity, which defines the gauge transform by means of which the generalised Coulomb gauge condition is realized. This gauge transform is uniquely determined by an exponential time-integral, the integration limits of which corresponds to infinitesimal time–period ϕ. The generalized coulomb gauge of Equation (11), if taken for granted for all time, makes generalized velocities time-dependent in relation to each other and in integral calculus, the time-dependent quantities can be treated as time-independent one during infinitesimal time-period ϕ. As such, during this infinitesimal time-period ϕ, the generalized velocities $\partial_0 A_k$ can be treated as independent quantities for all intent & purpose and can be used for the construction of short-lived canonical momentas in the Equation (3).

Accordingly, when these short-lived canonical momentas are substituted in the Equation (4), the resulting Hamiltonian H exists only for an infinitesimal time period ϕ because the implementation of the non-abelian Gauss law $A_0 = A_0\{A_k\}$ at Lagrangian level is only for this infinitesimal time period ϕ.

This implies that the physical applications, like working out the physical particle spectrum of QCD from this short-lived Hamiltonian H, can only yield short-lived massless gauge bosons. Further, although the dynamical time-dependent gauge freedom of the Hamiltonian H can be fixed by gauge fixing Equation (11) only for infinitesimal time period ϕ during which the non-abelian Gauss law $A_0 = A_0\{ A_k\}$ at Lagrangian level is implemented, but there still remains during this infinitesimal time-period ϕ the freedom of performing time-independent gauge transformations due to Gribov ambiguity. For studying this Gribov ambiguity, we refer to path-integral method in next section.

3. PATH INTEGRAL METHOD

We would now take the case of path integral method for quantization of classical Yang – Mills theory for strong interactions. In this path-integral method, the vacuum-to-vacuum transition amplitude is given by,

$Z = \int [dA] \exp. [i \int d^4x\, L(A)]$ where $\int d^4x\, L(A) = L'$ and $i = (-1)^{1/2}$ __________________ (12)

The gauge transformation is characterized with the invariance of the of the action $\int d^4x\, L(A)$. The very fact, that the above functional integral in Equation (12) of the vacuum-to-vacuum transition amplitude is to be evaluated over the space of the gauge potentials, compels one to take into account only gauge-inequivalent potentials A to preclude any possible multiple counting. Towards this end, we make use of the gauge fixing condition of Equation (11) above. After fixing the gauge, it is quite natural to question the extent of the uniqueness of the gauge-fixing scheme.

Gribov [9] in 1978 **mathematically formulated** that in non-abelian gauge theories, the gauge-fixing schemes like that of Equation (11) above are not unique. In the Yang – Mills theory, the gauge transformation is not global and is space-time dependent. In order to construct an action which includes derivatives and which is invariant under local gauge transformations, the gauge potential *A* transforms as a connection in the adjoint representation under local gauge transformation. Accordingly, in Yang – Mills theories, it is not the gauge potential themselves that are physical objects but their gauge orbits. The space of the orbits is usually parameterized by fixing the gauge. Here this can be done by a gauge fixing condition of Equation (11) above.

The uniqueness of the gauge fixing scheme, then, means that only one field configuration *A* in each orbit is allowed to satisfy the gauge condition. But Gribov in 1978 **mathematically demonstrated** that this uniqueness fails as the gauge condition Equation (11) is satisfied by two or more gauge-equivalent field configurations in each gauge orbit. This is called Gribov ambiguity and the additional gauge-equivalent field configurations satisfying the gauge condition in each gauge orbit are called Gribov copies. Here the "gauge orbit" for some configuration *A* is defined to be set of all gauge-equivalent configurations *A'*. Each point *A'* on the gauge orbit is obtained by transforming *A* as a connection in the adjoint representation under local SU(3) gauge transformation g as depicted below in Figure 1.[20]

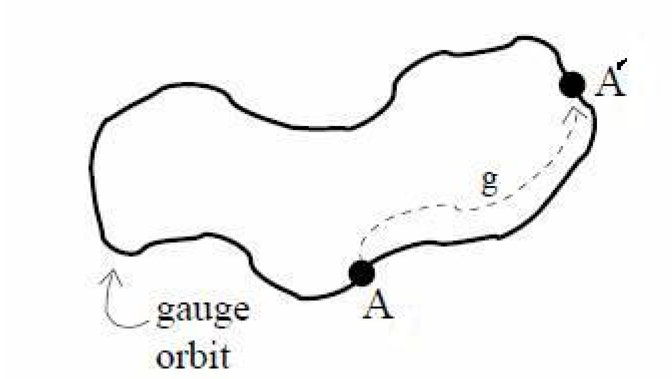


Figure.1 Illustration of the gauge orbit containing A and indicating the effect of acting on A with the gauge transformation g. The action $\int d^4x.L(A)$ is constant around the gauge orbit.[20]

"The integral space of the gauge fields in the integral $\int dA$ in the equation (12) above can be visualized as a product of integral length of a full set of gauge-inequivalent (i.e., gauge-fixed) configurations in the integral $\int dA^{g.f.}$ and integral length over the gauge group $\int d_g$. In other words, the integral length in the integral $dA^{g.f.}$ corresponds to the set of all possible gauge orbits and the same in the integral $\int d_g$ refers to the length of the gauge orbits. As such, we can conclude

$$\int dA \equiv \int dA^{g.f.} \int d_g$$

To make integrals such as those in the Eq. (12) finite and also to study gauge-dependent quantities in a meaningful way, we need to eliminate this integral around the gauge orbit, $\int d_g$.

The Faddeev-Popov gauge-fixing procedure eliminates this integral around the gauge orbit, $\int d_g$ in the perturbative QCD and in that way, it leads to ghosts and the local BRST invariance of the gauge-fixed perturbative QCD action. Since, for small field fluctuations of the perturbative QCD in the asymptotic regime, the Gribov copies [9] cannot be conscious of each other, so they can be neglected. But this situation does not the same in the non-perturbative QCD. Accordingly, the definition of the non-perturbative QCD should be such that the functional integral (12) contains each gauge orbit only once in order to eliminate the aforementioned integral around the gauge orbit, $\int d_g$. In other words, the non-perturbative QCD is to be defined in such a way that it has no Gribov copies. An implicit assumption in lattice QCD studies is to define the non-perturbative QCD in this way.

The generalized Faddeev-Popov technique is used for arriving at this definition of the nonpertubative QCD and in that way, just one gauge configuration on the gauge orbit is not chosen but rather what actually chosen is some Gaussian weighted average over the gauge fields on the gauge orbit. In the light of this choice, a non-local action $\int d^4x\ L(A)$ and a non-local quantum field theory arises in this definition of the nonpertubative QCD. But the proof of the renormalizability of QCD, the proof of asymptotic freedom, local BRST symmetry, and the Schwinger-Dyson equations etc. are obtained on the basis of this action $\int d^4x\ L(A)$. So, when this action $\int d^4x\ L(A)$ becomes non-local, these features of QCD cannot be proved in the non-perturbative context due to the absence of reliable basis.

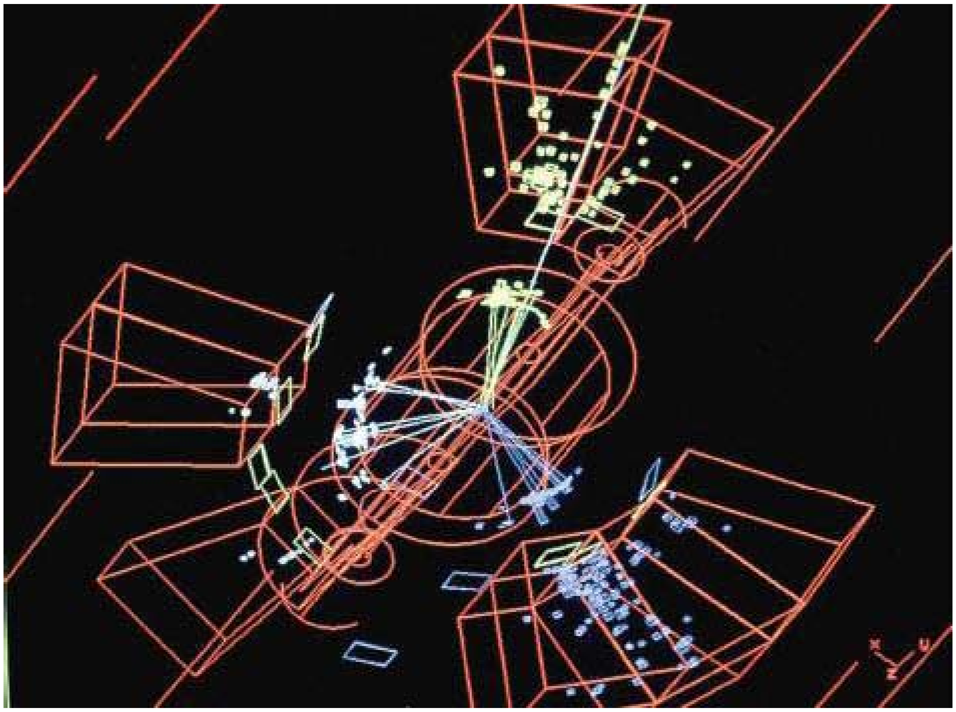
In other words, the basic features like locality and the BRST invariance of the QCD theory stand shattered in this definition of the nonpertubative QCD. But many authors [16 -19] in the literature have upheld an equally valid viewpoint that QCD must be defined with locality and BRST symmetry at the centre and these features should not be given up while defining QCD in the nonpertubative sector. The implications of this viewpoint are that the presence of Gribov copies is absolutely necessary, that the multiple representations of the Gribov orbits are a fact."[20]

In-fact, the aforementioned indispensable presence of the Gribov copies in the definition of nonperturbative QCD forces one to search for the physical interpretation of these Gribov copies. As depicted in the Figure.1 of [18] above, the "gauge orbit" for some configuration *A* is defined to be set of all gauge-equivalent configurations, and by definition, the action $\int d^4x\ L(A)$ is gauge invariant, so all the configurations *A'* on the gauge orbit have got same action.

This implies that the Gribov copies, in each gauge orbit, correspond to one and same physical situation such that there is residual gauge freedom given by gauge transformation between these Gribov copies. For exhibiting this residual gauge freedom in the light of the aforementioned indispensable presence of the Gribov copies in the definition of nonperturbative QCD, the necessary and sufficient condition is that the representative gauge potential on each of the gauge orbit in the physical configuration space (the space of all gauge orbits) must be surrounded by all the Gribov copies of that gauge orbit in Minkowski space-time. In other words, any real gluon as a representative gauge potential on each of the gauge orbit in the physical configuration space must be surrounded by massless gauge bosons, each referring to one of the Gribov copies of that gauge orbit in Minkowski space-time.

6. DISCUSSION:

The above conclusion implies that a real gluon, created as one of three jets emerging from electron-positron annihilation at high energy in following photograph from L3 Collaboration, CERN [21] at any point of time, would be immediately surrounded by its Gribov copies in the QCD vacuum.



In other words, this real gluon and its Gribov copies in QCD vacuum are multiply representing a particular gauge orbit that exists in the space of gauge potentials. If we envisage the gauge field of this particular gauge orbit, that is multiply represented by the real gluon and its Gribov copies, being used to perturbatively dress the matter field of a perturbative static Lagrangian quark, we can easily construct gauge-invariant colored static quark in perturbation theory for the mere fact [20] that here standard gauge fixing is unique for a gauge orbit as Gribov copies cannot be aware of each other for small field fluctuations and can be easily neglected for large momenta. Indeed, classical QCD is scale-invariant as there are no natural scales in it.

But a confining scale automatically gets generated in non-perturbative QCD at such a spatial locality when the Gribov copies, representing the same gauge orbit, become aware of each other and start interchanging their representative role by exhibiting residual gauge transformation amongst them. At this spatial locality, we can cope with the insensitivity [22] of the dressing (of gauge orbit) towards matter field color (of static

Lagrangian quark) at the place of occurrence of the Gribov copies on this particular gauge orbit by postulating different color charges to the Gribov copies so that the overall color charge of the matter field and dressing is preserved on the whole of this particular gauge orbit.

Since, there is no inherent preference for any specific color charge so far as the representation of that particular gauge orbit by real gluon and its Gribov copies in QCD vacuum is concerned, so, it is perfectly straightforward to consider the ensemble of the real gluon and its Gribov copies in QCD vacuum as color-singlet one. Further, this color-singlet ensemble of the real gluon and its Gribov copies is physically co-existent in QCD vacuum as multiple representatives of the same gauge orbit, so, the static matter field, that is being dressed with the gauge field of this gauge orbit, also needs to be color-singlet one. This explains why the matter field of colored quark is forbidden to be dressed with the gauge field of the gauge orbit in non-perturbative QCD and the color-singlet hadron is allowed as physical observables after combined matter field of its constituents is dressed as a whole with the gauge field of the gauge orbit in non-perturbative QCD. This is nothing but confinement mechanism of quarks in non-perturbative QCD.

Next, the question arises how mass gap is generated in the above scenario. In pure gluodynamics, the above explained color-singlet ensemble of the real gluon and its Gribov copies is physically co-existent in QCD vacuum as multiple representatives of the same gauge orbit. At perturbative level, the Gribov copies, as already stated, are not aware of each other and consequently, all the real gluons remain massless. However, this normal massless vacuum is unstable, since each and every real gluon and their corresponding Gribov copies become aware of each other and start interacting with each other to form color-singlet ensemble called Gribov glue ball at such a spatial locality when confining scale automatically gets generated at non-perturbative level.

Consequently, stable vacuum is realized after the condensation of these Gribov glue balls. The excitation spectrum has a dynamical mass gap generation because the constituent real gluon and its Gribov copies in any color-singlet Gribov glue ball continuously exhibit residual gauge transformation amongst themselves by interchanging their respective roles as multiple representatives of the common gauge orbit. The amount of dynamical mass gap generated in any Gribov glue ball depends upon the largeness of gauge field that is carried by its respective gauge orbit.

Lastly, the mechanism for phase transition from confinement to deconfinement with increasing temperature needs to be probed in above scenario of Gribov glue balls. In pure gluodynamics, the confined phase and deconfined phase are distinguished by an order parameter called the expectation value of the Polyakov loop [23] that is zero for the confined phase and non-zero for the deconfined one. Naturally, Gribov glue balls exist in confined phase. From symmetry point of view, the Polyakov loop characterizes the breaking of Gribov glue balls. At critical temperature, the real gluon and its Gribov copies starts losing contact with each other for small field fluctuations and this ultimately causes disassociation of Gribov glue balls above critical temperature when phase transition from confined phase (of Gribov glue balls) to deconfined phase (of gluon plasma) takes place.

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