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Surfing Momentum, Mass, Energy and Dark Matter

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Abstract

The datasets by Bertozzi [9] and by Lund and Uggerhøj [10] show that an electron with kinetic energy (normalised with respect to its rest energy) near to or greater than 2 is underestimated by Einstein's generalised formula for the energy of a moving object/particle. Also, the datasets show that the highest kinetic deficit in Einstein's formula occurs at the data point with the highest kinetic energy or highest speed. This energy deficit or anomaly becomes evident when energy is plotted again speed. It is not so evident vice versa, as is often the case; hence the anomaly was not identified. A new definition of energy incorporating the surfing momentum of a particle on its phase (S) surface is proposed. This new definition predicts or explains the observed energy deficit in Einstein's generalised formula and can further our understanding of the nature of rest mass, rest energy and massless particles. Using the criterion of least squares, it is shown that the new formula performs better than Einstein's formula in representing the relationship between kinetic energy and speed when data points with normalised kinetic energy near to or greater than 2 are included. It is suggested that more experimental data should be gathered to further compare the performances of Einstein's formula and the new formula. If the new formula for energy is validated by further experiments, this could shed some light on the puzzle of dark matter because a pair of supposedly annihilated particle and anti-particle can continue to exist together as a joint entity in a 'massless' state with zero spin and zero charge, but with non-zero kinetic energy according to the new formula for energy (Einstein's formula will give zero kinetic energy for such a massless entity). These joint entities with non-zero kinetic energy could be dispersed in the vast space of a galaxy and could at least partially account for the elusive dark matter. According to the new formula, the energy released in such an 'annihilation' is twice the rest energy of the particle, as is generally understood. The 'creation' of a pair of particle and anti-particle can be understood as the reverse of the 'annihilation' process. Because of such implications of the new formula, gathering further experimental data to compare the performances of the new formula and Einstein's formula is an important task. Also, the new formula, if validated, will add credibility to the relativistic model for quantum particles (expounded in [8]) on which the new formula is based. Finally, a nuanced understanding of rest mass and rest energy based on the new formula and its underlying relativistic model could have implications for our understanding of strong nuclear force.

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1.0 Introduction

Physicists have been looking for the dark matter necessary to hold the galaxies together gravitationally. Also, dark energy is necessary to account for the increasing inflation rate of the universe. As dark matter can also be considered in terms of energy, this energy and the dark energy are the two kinds of energy which have been elusive to physicists. In view of this, any useful additional insight into the nature of mass and the nature of energy should be welcome.

Hestenes wrote a paper on 'The *Zitterbewegung* Interpretation of Quantum Mechanics' [1] which encouraged other physicists such as Salesi, Recami and Espositio to write papers [2,3,4] along that line. These papers give some indication what particle spin could be. Hestenes hypothesised in Section 4 of his paper, 'The so-called "rest mass" of the electron is therefore a kinetic energy of the magnetic self-interaction. It is this that gives the electron its inertial properties.' He also referenced the 'flywheel-like nature of this inertia' but he did not investigate the dynamics of such inertia in that paper. More recently, he again suggested in his essay on 'Electron time, mass and zitter' [5], 'The spin-zitter hypothesis has implications for gravitational fields as well as sources. It tells us that there is no mass without spin.' In a paper on 'Spin and Relativity', Lepadatu [6] also hypothesised, 'The inertia is an intrinsic property due to the spin motion of the particles, ...' Rockenbauer [7] referenced Hestenes and Lepadatu in his paper which is entitled 'Can the spinning of elementary particles produce the rest energy mc^2? The vortex model of elementary particles'. He also hypothesised, '[T]he rest energy can be produced in full by the spinning motion of elementary particles if the peripheral speed is equal to the velocity of light.' The challenge with these hypotheses is that they rely on the particle moving at the speed of light. This paper will take a significantly different route to the routes indicated by these authors and will not require a particle to move at the speed of light. It will make use of the idea that the rest mass is generated from the inherent mass of a particle by some suitable surfing motion on the phase surface – the surfing motion incorporates the spinning motion of the particle but has an additional velocity component. Hence, contrary to Rockenbauer, the paper will suggest that the surfing energy of the particle on the phase surface, which includes but is greater than its spin energy, is responsible for the rest energy of the particle.

The paper will briefly introduce the relativistic model for a particle and its spin which has been given extensive treatment by the author in a previous paper [8]. The focus of this paper is on the implications of the relativistic model for our understanding of the mass and energy of a particle. Briefly, in the model, because of the additional surfing motion of the particle on its phase surface which is perpendicular to its translational motion, the total speed of a particle is always greater than its translational speed. This implies that the Lorentz factor in the model is always greater than the one conventionally given merely by the translational speed. Hence, the conventional expression for the energy of a particle, given by the smaller conventional Lorentz factor, invariably yields an energy below that given by the larger Lorentz factor in the model adopted here (see later). A new definition of energy will be presented incorporating the larger Lorentz factor. This new definition of energy corresponding to the adopted model ought to be verified or falsified by experimental data. If indeed experimental data favour the new definition of energy compared to the conventional definition of energy, it will strengthen the credibility of the model, which incidentally is already consistent with a significant number of observed phenomena (see sections 10.1 and 10.2 of [8]). It will be shown in this paper that there are two sets of existing experimental data which favour the new definition of energy. One set of data came from the experiment performed at MIT while the other set of data came from the experiment performed more recently at the University of Aarhus, Denmark. However, because of the weightiness of the matter, the paper will suggest that more experiments should be performed to verify or falsify the new definition of energy and its associated relativistic model. If verified, the new definition will have implications for our understanding of dark matter and other matters in physics.

2.0 Surfing Motion of a Particle on Its Phase Surface

The relativistic model for a particle is given in section 8 of [8]. Here it is presented in a highly summarised form. A particle has three momentum components:

$$\underset{\sim}{p}_1 \equiv \hbar \nabla S \ , \ \underset{\sim}{p}_2 \equiv \lambda_2 \frac{\hbar}{2} \frac{\nabla \rho}{\rho} \wedge \vec{s} \ , \ \underset{\sim}{p}_3 \equiv \lambda_3 \hbar \frac{\nabla S}{\rho} \wedge \vec{a}$$

where the wave function is written as $\psi = Re^{iS}$, $\rho \equiv R^2$ is the probability density,

$\vec{s}$ at the particle's position is a unit vector perpendicular to $\nabla\rho$ and lies on the plane formed by ∇S and $\nabla\rho$, $\vec{a}$ is the unit vector in the direction of $\underset{\sim}{p}_2$ at the point where the particle is, λ_2 is a constant corresponding to the spin number of the particle in question, λ_3 is constant over space but varies in time non-deterministically. The three momentum components form an orthogonal set of vectors and they correspond to the three velocity components, $\underset{\sim}{v}_1, \underset{\sim}{v}_2, \underset{\sim}{v}_3$, which also form a set of orthogonal vectors. $\underset{\sim}{v}_1$ is called the translational velocity. $\underset{\sim}{v}_2$ and $\underset{\sim}{v}_3$ lie on a plane tangential to the *S* (phase) surface at the position where the particle is. This means that $\underset{\sim}{v}_s \equiv \underset{\sim}{v}_2 + \underset{\sim}{v}_3$ is a velocity on that tangential plane so that the particle can be said to be surfing on the *S* surface while moving forward with velocity $\underset{\sim}{v}_1$. Hence, $\underset{\sim}{v}_s$ is called the surfing velocity.

It will be instructive to illustrate these three orthogonal velocity components with the case of a free particle with no slit in its path to diffract it. The following Helmholtz equation in *R* applies both in the relativistic framework and the non-relativistic framework:

$$\nabla^2 R + a^2 R = 0$$

where *a* is a constant.[2] (This implies that in free space with no slit, the quantum potential, $-\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$ is constant.) In a system of cylindrical co-ordinates, (*r, θ, z*), we adopt the convention that $\underset{\sim}{v}_1$ is in the *z* direction. It can be easily shown that a *S* surface is then identical to the (*r, θ*) plane (or the *z* plane) on which the particle surfs non-deterministically. It can also be shown that the *R* surfaces, and therefore the *ρ* surfaces, are circular tubes extending along the direction of *z*; and *R*=0 at a certain distance, *L*, from the centre.[3] Some sample circular *R* contours and the surfing velocity components, $\underset{\sim}{v}_2$ and $\underset{\sim}{v}_3$, are illustrated in Figure 1. Again, $\underset{\sim}{v}_3$ varies non-deterministically but will be subjected to the bulk statistical constraint of Born's rule (see below).

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[2] See (§7) in [8] for the non-relativistic case and (§22) for the relativistic case.

[3] See section 4.0 of [8] for the mathematical details which also apply to the Helmholtz equation of the same form for the relativistic case, i.e., (§22) of [8].

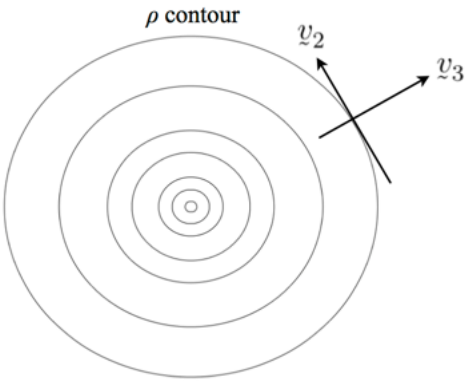

Figure 1: $\underset{\sim}{v}_2$ and $\underset{\sim}{v}_3$ on the (r, θ) plane (or S surface); the z direction and hence the direction of $\underset{\sim}{v}_1$ is perpendicular to the page

The choices of the forms of the three momentum components are justified by correspondence to observational data as follows. Firstly,

$$\left|\underset{\sim}{p}_1\right| \equiv \hbar|\nabla S| = \hbar\frac{2\pi}{\lambda} = \frac{h}{\lambda}$$

reproduces de Broglie's formula relating the momentum of a particle to the 'wavelength' of a particle. Hence, the choice of $\underset{\sim}{p}_1$ is consistent with (or vindicated by) the observed de Broglie's relation. Secondly, in the free particle case, integrating the angular momentum, given by the product of $\underset{\sim}{p}_2$ (the spinning momentum) and the radial distance, over the domain where the particle can be found yields $\lambda_2\hbar$, which is the angular momentum of the particle and is constant. Hence, the choice of the expression for $\underset{\sim}{p}_2$ is consistent with (or vindicated by) the observed constant angular momentum of a particle (see section 8.3.1 of [8] for details). Thirdly, $\underset{\sim}{p}_3$ is prescribed as the non-deterministic momentum through the non-deterministic λ_3 in such a way as to satisfy the observed Born's rule and the observed non-determinacy of a particle. Hence, all three prescribed momenta match the observations in our universe and in that sense they are credible. In the philosophy of science, a theory's credibility is best assessed by its correspondence to observation, even though the criterion of elegance can be a supplementary criterion (to some degree the latter is true for the set of three orthogonal momenta). In the sense of correspondence to observation and in the sense of elegance, the three prescribed momenta are credible and will be used in this paper.[4]

Corresponding to the surfing velocity on the *S* surface, $\underset{\sim}{v}_s$, the surfing momentum is defined as

$$\underset{\sim}{p}_s \equiv \underset{\sim}{p}_2 + \underset{\sim}{p}_3$$

which will be a significant entity in our consideration of rest mass and rest energy.

3.0 'Rest Mass', 'Rest Energy', Einstein's Energy Formula and a New Energy Formula

Einstein's well known energy formula is

$$E = mc^2$$

which can be applied to a particle with zero translational velocity. Note that, this formula does not involve the notion of a surfing momentum as this paper suggests. Hence, this formula assumes that the particle is completely at rest, *m* is the rest mass and the energy, *E*, is the rest energy.

For non-zero translational velocity with speed v_1, *E* can be written as

$$E = m_r c^2$$

where m_r is the relativistic mass, $m_r = m/\sqrt{1 - v_1^2/c^2}$. In this paper, we call this energy formula as Einstein's generalised energy formula, or simply as Einstein's formula. This energy formula can be re-written as

$$E^2 = p_{1E}^2 c^2 + m^2 c^4 \qquad (\S1)$$

[4] See section 10.1 of [8] for a summary of how these three velocity components are consistent with a significant number of other observed phenomena which thus add credibility to them.

where p_{1E} is the magnitude of the momentum as Einstein envisaged it:

$$p_{1E} \equiv \frac{mv_1}{\sqrt{1-\frac{v_1^2}{c^2}}} \qquad (\S 2)$$

where again v_1 is the translational speed and the surfing motion perpendicular to $\underset{\sim}{v}_1$ is not involved in the expression for the magnitude of this momentum which is thus called the translational momentum. These expressions for the magnitude of the momentum and the energy are the conventional expressions. The subscript '*E*' in p_{1E} signifies its expression according to Einstein and the subscript '1' signifies that it is the translational momentum.

The surfing motion and the associated surfing momentum of the particle ought to be represented and included in the expression for energy in (§1). But how is this inclusion possible? Since the first term on the r.h.s. of (§1) comes from the translational momentum, it is logical that the surfing momentum could be related to the second term in (§1), m^2c^4. And if we set the magnitude of the surfing momentum as $p_s = mc$, then (§1) becomes $E^2 = p_{1E}^2c^2 + p_s^2c^2$. This looks like a balanced expression with the translational momentum and the surfing momentum both contributing to the energy. Furthermore, if we denote the magnitude of the total momentum by *p*, then $p^2 = p_{1E}^2 + p_s^2$ and $E = pc$. But now since the total speed is no longer merely v_1, the Lorentz factor for the translational momentum should no longer be that in (§2). Furthermore, the definition of the magnitude of the surfing momentum, p_s(see (§6)), involves a mass parameter which is not necessarily the same as the rest mass *m*, that is, it is possible that the rest mass, $m = p_s/c$, is generated by p_s which incorporates a different and a more fundamental mass in its expression.[5] The larger Lorentz factor and the possibility of a more fundamental mass suggest that we should begin with a fresh and a more radical basis which will give consistency and elegance to the

[5] Contrary to Hestenes, Lepadatu and Rockenbauer, this paper suggests that the rest mass is generated by the surfing momentum which includes but is more than the spinning momentum of the particle.

forms of translational momentum, surfing momentum and energy. Nevertheless, the above intuitive exercise has led us to see the possible relationship between the surfing momentum and rest mass. (For the sake of brevity, from now on whenever the term ‘momentum’ is used, unless otherwise stated, it means the magnitude of the corresponding momentum.)

Now, we define a new energy with subscript *N* (for new) to distinguish it from *E* given by (§1), a new total momentum and a new translational momentum:

$$E_N \equiv pc \qquad (§3)$$

$$p \equiv \frac{m_i v}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (§4)$$

$$p_{1N} \equiv \frac{m_i v_1}{\sqrt{1-\frac{v_1^2+v_s^2}{c^2}}} \qquad (§5)$$

where v is the total speed, $p = \left|\underset{\sim}{p}\right|$, $\underset{\sim}{p} = \underset{\sim}{p}_{1N} + \underset{\sim}{p}_s$, $\underset{\sim}{p}_s = \underset{\sim}{p}_2 + \underset{\sim}{p}_3$, m_i is the ‘inherent mass’ of the particle. The term ‘inherent mass’ means the mass inherent to the particle which does not arise from any motion of the particle, not even its surfing motion on the *S* surface. $\underset{\sim}{p}$ is the total momentum (vector) of the particle taking into account both the translational momentum *and* the surfing momentum on the *S* surface. Similar to the total momentum in (§4) and the translational momentum in (§5), the surfing momentum is consistently defined as

$$p_s \equiv \frac{m_i v_s}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_i v_s}{\sqrt{1-\frac{v_1^2+v_s^2}{c^2}}} \qquad (§6)$$

where v_s is the surfing speed in general. These definitions of energy and momenta consistently use the same Lorentz factor which include the surfing speed, v_s, and the same inherent mass, m_i. Taking the square of (§3), we have

$$E_N^2 = p^2c^2 = p_{1N}^2c^2 + p_s^2c^2 \ . \quad (§7)$$

If we compare (§7) with (§1), we see that m^2c^4 could be equivalent to and therefore accounted for by $p_s^2c^2$ such that

$$p_s = mc \ . \quad (§8)$$

The 'rest mass', *m*, is given by $m = p_s/c$. A constant *m* requires a constant p_s. Hence, the variation in the translational motion of a particle should not affect the magnitude of the surfing momentum. This is reasonable since the surfing motion on the *S* surface is perpendicular to the translational motion. However, as the translational speed v_1 varies, the total speed v varies, v_s will need to adjust to maintain a constant p_s according to (§6).

In the particular case when the translational speed is zero, v_{so} is the surfing speed and

$$p_s = \frac{m_i v_{so}}{\sqrt{1 - \frac{v_{so}^2}{c^2}}} \quad (§9)$$

The generated 'rest mass' , which is p_s / c, can be set by using this expression for zero translational speed. At this point it will be good to begin to use the term, 'effective mass', rather than the term, 'rest mass', which can be misleading since the particle is not at rest due to the surfing motion on the *S* surface even when its translational speed is zero. From now on the term 'effective mass' and the term 'rest mass' (if it is used at all), denoted by *m*, will have the same meaning which is nevertheless different from the meaning of 'inherent mass', m_i. Again for zero v_1, both p_{1E} and p_{1N} are zero and $E_N = p_s c = E = mc^2$, which is normally called the 'rest energy' but is also called the 'base energy' in this paper (since the particle is not genuinely at rest with its surfing motion).

Note that, for zero $\underset{\sim}{v}_s$ and therefore zero p_s, the effective mass, *m*, will be zero even though its inherent mass, m_i , is not zero. This brings to mind the photon and other 'massless' particles whose effective mass is considered to be zero. In the model adopted *in this paper, these particles are 'massless' in the sense that their effective mass is zero, but zero effective mass does not exclude the possibility of these 'massless' particles having non-zero inherent mass*. The notion of 'massless' particle could be useful in understanding particle and anti-particle annihilation where the energy released in the annihilation is twice the base energy (rest energy) of the two particles. This could be understood in terms of the particle and anti-particle giving up their surfing momentum and their associated surfing energy – which is twice the base energy (rest energy) – and becoming a joint 'massless' entity (with zero effective mass) which nevertheless has inherent mass. This will be considered further in the section on Discussion. Also, according to (§5), such a 'massless' entity can have non-zero translational momentum if its inherent mass and its translational speed are non-zero; and it can have non-zero energy which is given by

$$E_N = p_{1N} c \qquad (\S 10)$$

according to (§7). The notion of non-zero energy for a 'massless' entity formed from annihilation could, at least partially, account for the elusive dark matter and will be discussed further also in the section on Discussion.

4.0 Comparison Between Einstein's Energy Formula, the New Energy Formula and Observed Data

If v_1 is greater than zero,

$$\frac{p_{1E}}{p_{1N}} = \frac{m}{m_i} \frac{\sqrt{1 - \frac{v_1^2 + v_s^2}{c^2}}}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{m}{m_i} \frac{\sqrt{c^2 - (v_1^2 + v_s^2)}}{\sqrt{c^2 - v_1^2}} \qquad (\S 11)$$

where it can be seen that p_{1N} will tend to be larger than p_{1E} when v_s is greater than zero. This is due to the crucial fact that the Lorentz factor in p_{1N} takes into account the effect of the surfing velocity on the *S* surface while the Lorentz factor in p_{1E} does not. The Lorentz factor in p_{1N} is naturally greater than the Lorentz factor in p_{1E} which lacks the surfing speed in its definition. However, the ratio between the p_{1E} and p_{1N} also depends on the ratio between the effective mass and the inherent mass. Now,

$$\frac{m}{m_i} = \frac{p_s}{m_i c} = \frac{m_i v_s}{m_i \sqrt{c^2 - v^2}} = \frac{v_{so}}{\sqrt{c^2 - v_{so}^2}} \ . \qquad (§12)$$

For $v_{so} = \dfrac{c}{\sqrt{2}}, \ m = m_i.$

For $v_{so} > \dfrac{c}{\sqrt{2}} \ , \ m > m_i.$

For $v_{so} < \dfrac{c}{\sqrt{2}} \ , \ m < m_i.$

The ratio between the p_{1E} and p_{1N} will affect the the ratio between *E* and E_N . Since m^2c^4 of (§1) is identical to $p_s^2 c^2$ of (§7), the difference between E^2 and E_N^2 lies in the magnitudes of p_{1E} and p_{1N}. If p_{1N} is greater than p_{1E}, then E_N will be greater than *E*. In that case, since E_N and *E* have the same base energy (rest energy), the kinetic energy in E_N, defined as the difference between E_N and the base energy, will be greater than the kinetic energy in *E*. To investigate whether this is the case, we will plot the graphs relating the kinetic energy of a particle to its translational speed for various values of $\dfrac{m}{m_i}$. Before we can do that, we need to work out the expressions for the kinetic energy in E_N and the kinetic energy in *E*.

For the Einstein case, the particle's kinetic energy is $E - mc^2 = m_r c^2 - mc^2$ where $m_r = m/\sqrt{1 - v_1^2/c^2}$ is the relativistic mass. It can be easily shown that, the kinetic energy (written as *K.E.*), when normalised with respect to its base energy (rest energy), can be written as

$$\frac{K.E.}{mc^2} = \left(1 - \frac{v_1^2}{c^2}\right)^{-\frac{1}{2}} - 1 \qquad (§13)$$

where it can be seen that the first term on the r.h.s. is the Lorentz factor.

For the case of the new definition of energy, the particle's kinetic energy (*K.E.*) due to its translational motion is

$$K.E. \equiv E_N - mc^2 = E_N - p_s^2/m$$

which, when normalised with respect to its base energy, is

$$\frac{K.E.}{mc^2} = \frac{E_N}{mc^2} - 1 = \left(\frac{p_{1N}^2 c^2 + p_s^2 c^2}{m^2 c^4}\right)^{\frac{1}{2}} - 1 = \left(\frac{p_{1N}^2}{p_s^2} + 1\right)^{\frac{1}{2}} - 1$$

where $\frac{p_{1N}^2}{p_s^2} = \frac{(v_1/c)^2}{(v_s/c)^2}$ according to (§5) and (§6). Using (§6) and (§8),

$$\left(\frac{v_s}{c}\right)^2 = \left(1 - \frac{v_1^2}{c^2}\right) \Big/ \left(1 + \frac{m_i^2}{m^2}\right)$$

which incidentally shows that $\left(\frac{v_s}{c}\right)^2$ decreases linearly with $\frac{v_1^2}{c^2}$ and approaches zero as $\frac{v_1^2}{c^2}$ approaches 1. Using the above relationships,

$$\frac{K.E.}{mc^2} = \left(1 + \frac{\frac{v_1^2}{c^2}}{(1 - \frac{v_1^2}{c^2})/(1 + \frac{m_i^2}{m^2})}\right)^{\frac{1}{2}} - 1\,, \qquad (§14)$$

where $\frac{m_i^2}{m^2}$ is an additional parameter that (§13) does not have in Einstein's case. It can be seen that when $\frac{m_i^2}{m^2}$ is zero, (§14) reduces to (§13), i.e., zero $\frac{m_i^2}{m^2}$ corresponds to Einstein's case. But other than this special case, $\frac{m_i^2}{m^2}$ is non-zero.

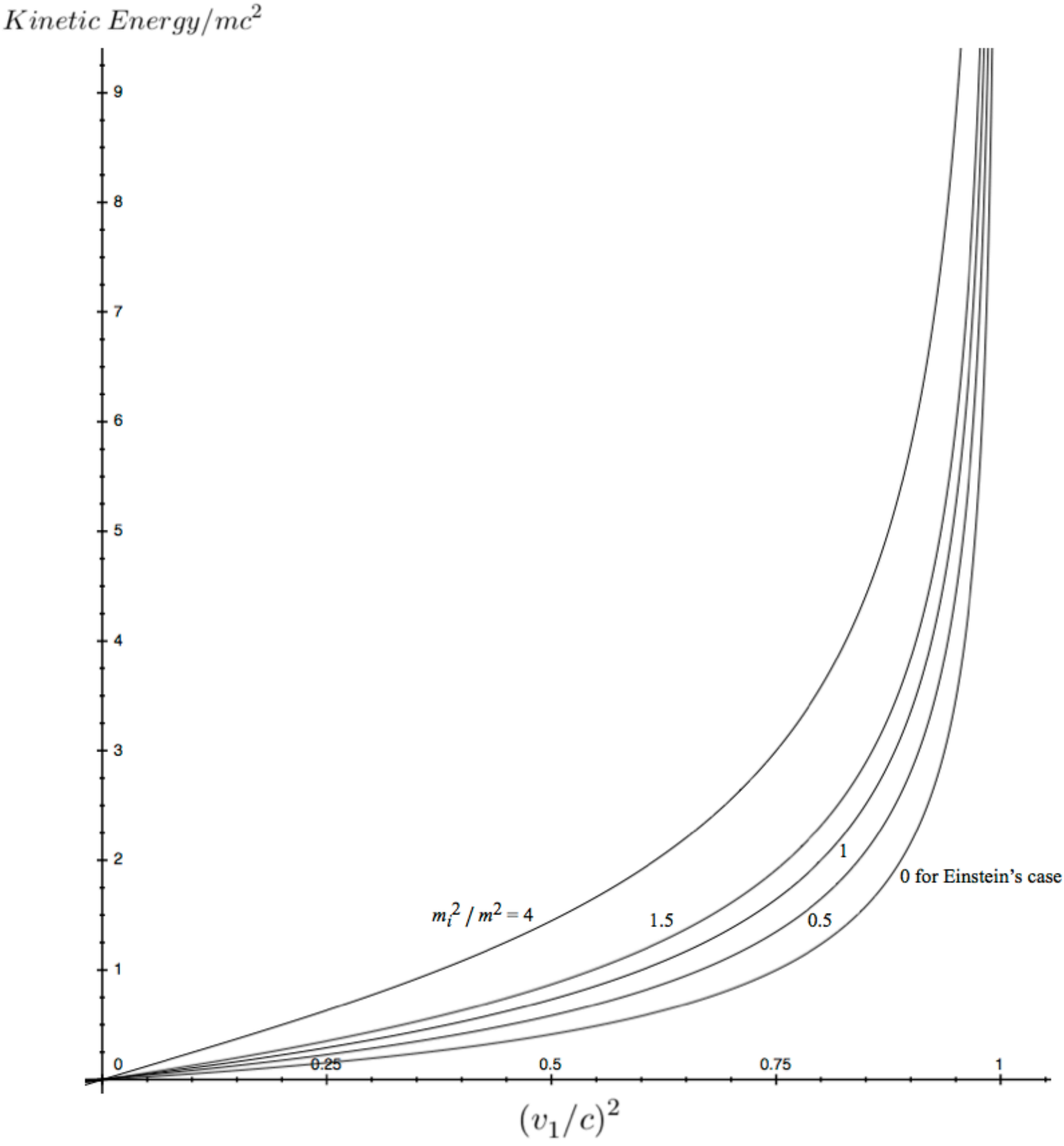

Figure 2: $Kinetic\ Energy/mc^2$ vs $(v_1/c)^2$ with parameter value for $m_i^2 \ / \ m^2$ as 4, 1.5, 1, 0.5 and 0 (Einstein's formula)

It is readily seen that as $m_i^2 \ / \ m^2$ increases from zero, the curve shifts to the left in Figure 2. This means that for any given translational velocity, the kinetic energy according to Einstein's formula is invariably less than the kinetic energy (due to the translational motion, not the surfing motion) according to the new formula. And the difference in kinetic energy between the two formulae for a given translational speed, $\triangle(Kinetic\,Energy/mc^2)$, increases with increasing value of $m_i^2\ /\ m^2$. Also, for a given parameter value of $m_i^2\ /\ m^2$, the difference in kinetic energy between the two formulae will tend to infinity as the translational speed, v_1, approaches c, as can be seen in the following graph.

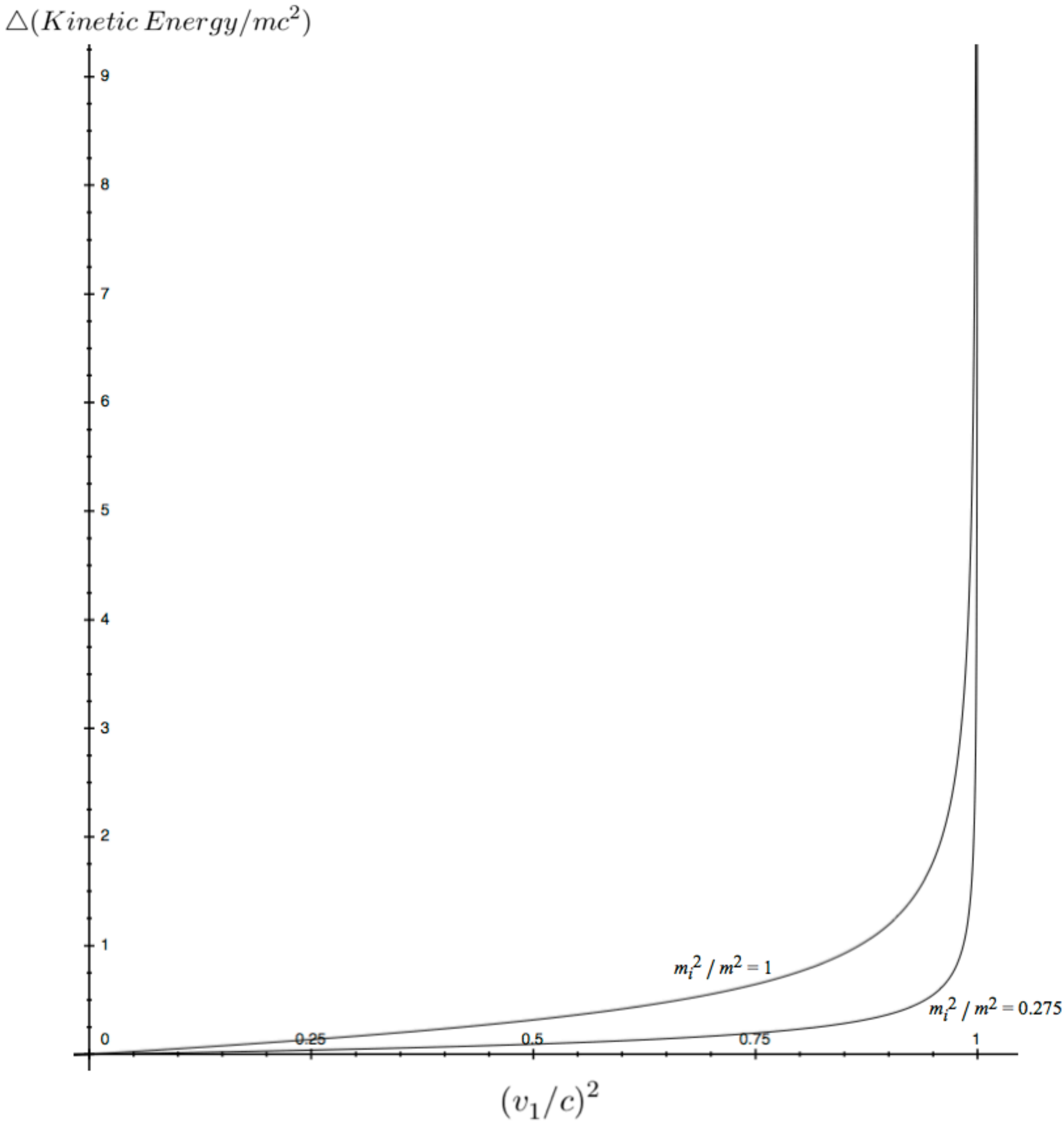

Figure 3: Difference in kinetic energy (normalised by rest mass) between Einstein's formula and the new formula , $\triangle(Kinetic\,Energy/mc^2)$, vs $(v_1/c)^2$, with parameter value for $m_i^2\ /\ m^2$ of the new formula as 1 and 0.275.

Since for a given non-zero translational velocity, the kinetic energy of a particle according to the new formula will be higher than that according to Einstein's formula, an experiment correlating the translational speed of a particle with its kinetic energy can therefore decide which formula is closer to observation. In fact, such experiments have already been performed. Bertozzi [9] carried out such an experiment at MIT with electrons in the 1960s; Lund and Uggerhøj [10], two physicists from Aarhus University, Denmark, also performed a similar experiment with electrons in 2009. Their experimental results will now be discussed. Figure 4 contains the plot of Bertozzi's result.

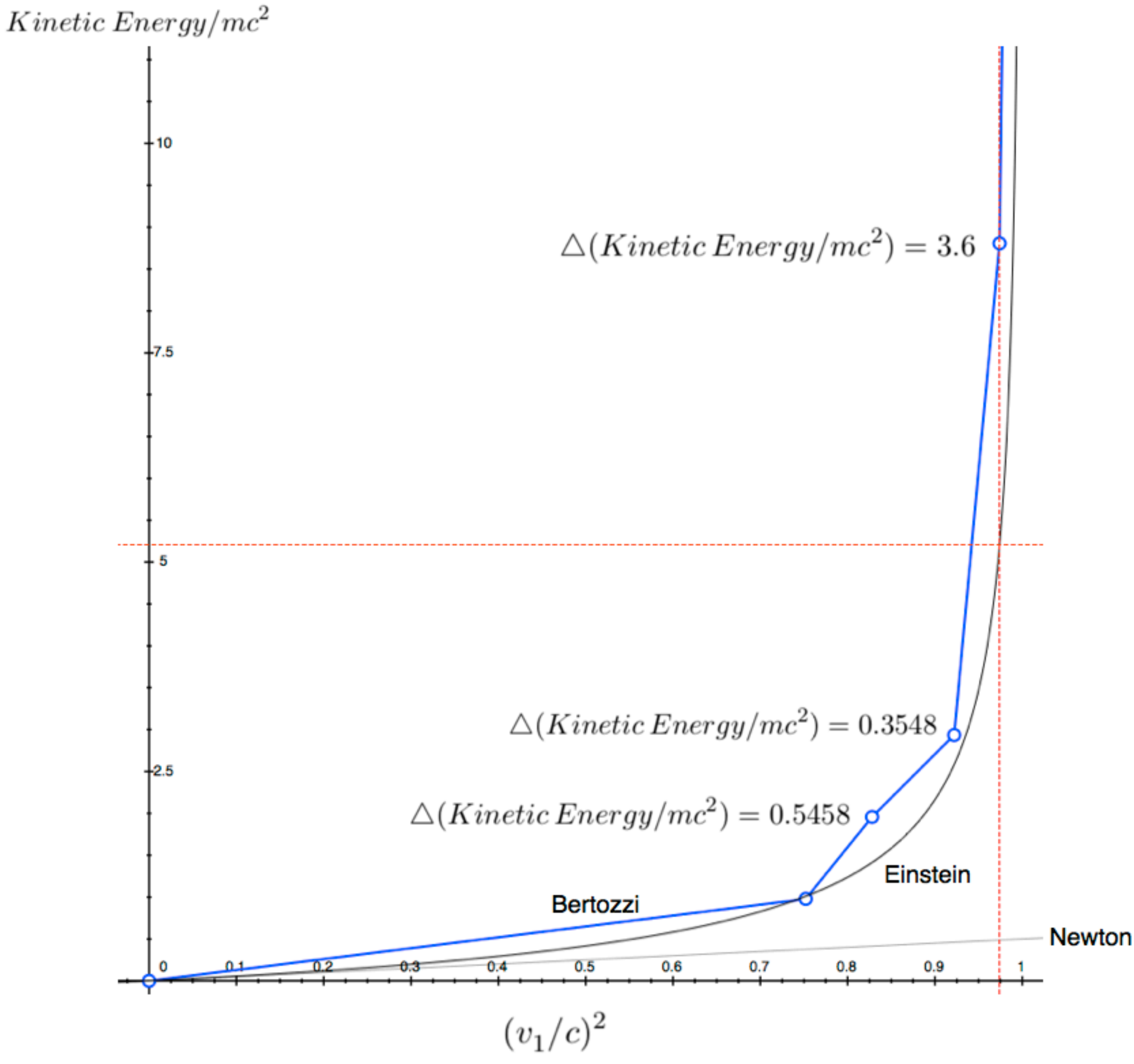

Figure 4: $Kinetic\ Energy/mc^2$ vs $(v_1/c)^2$ for Einstein, Bertozzi and Newton

Data Point	$(v_1 / c)^2$	K.E. (Mev)	K.E. / mc^2	K.E. / mc^2 according to Einstein	$\triangle$K.E. / mc^2
B1	0.752	0.5	0.9785	1.008	-0.0295
B2	0.828	1	1.957	1.4112	0.5458
B3	0.922	1.5	2.9354	2.5806	0.3548
B4	0.974	4.5	8.8063	5.2	3.6063

Table 1: The data from Bertozzi's experiment,[6] compared with prediction from Einstein's formula

Bertozzi's experimental data follow the general trend of the curve for Einstein's formula but we can see from his data that there is an anomaly, i.e., apart from the data point with the lowest speed, at high speed Einstein's formula consistently underestimates the kinetic energy of the electron, which is predicted by the new energy formula proposed in this paper; see the three last values of $\triangle(Kinetic\,Energy/mc^2)$ at B2, B3 and B4 which give the differences between Bertozzi's data and the values given by Einstein's formula. The greatest energy deficit marked by the red dotted lines in Figure 4 (for data point B4) is 3.6 times of the electron's rest energy (or base energy) which is a very large amount of energy for the electron. The other two energy deficits, for data points B2 and B3, are 0.5458 and 0.3548 times of the electron's rest energy (or base energy) which are considerable amounts for an electron. Because Bertozzi was looking to see whether the data follow the trend of the curve corresponding to Einstein's relativistic formula or the curve corresponding to Newton's non-relativistic formula, he had his conclusive answer to this question, i.e., Einstein's formula is much better than Newton's formula. However, he failed to see that for high speed there are consistent and significant energy deficits in Einstein's formula compared to his observed data. Perhaps, he might have put it down to some experimental errors in measuring the

[6] The last data point from Bertozzi is not used in the plot. For that data point, the kinetic energy is so large that the translational speed is very close to *c* such that he set it to be equal to *c*. But that requires infinite kinetic energy, not a finite large kinetic energy. This means that accurately measuring the translational speed so close to *c* is beyond the capacity of his experiment. Hence, that data point is not used in the plot in Figure 4.

electron's speed. However, *the observed data for high speed electrons (B2, B3 and B4) consistently fall on only one side of Einstein's curve, as the new formula predicts*. And *the kinetic energy deficit becomes evident at high speed, which the new formula also predicts* (see Figure 3). Bertozzi did not discuss the anomaly of the deficits in kinetic energy according to Einstein's formula probably because he was interested in his original question concerning Einstein's formula and Newton's formula, and nothing more. At this point, one needs to be cautious not to arrive at any premature conclusion since only four data points from Bertozzi's experiment have been used. More experimental data are necessary to clarify the energy and speed relationship.

Lund and Uggerhøj [10] also performed a similar experiment with electrons in 2009 as Bertozzi did in 1964 but with smaller kinetic energies. Here is their result.

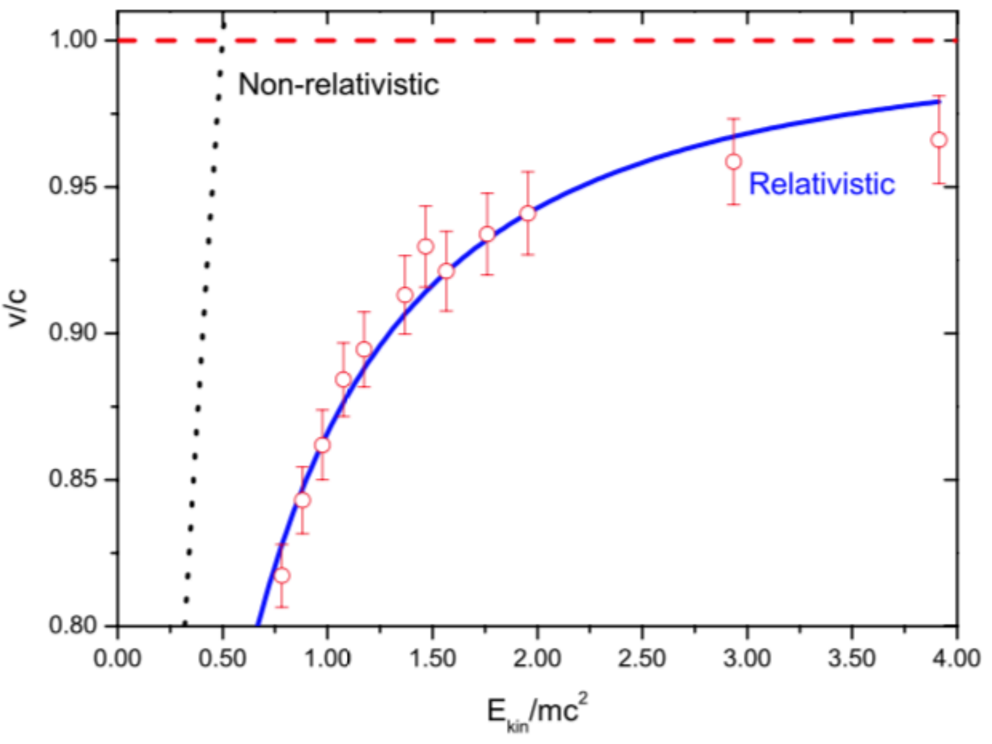

Figure 5: v_1/c (normalised translational speed, *v/c* in graph) vs $Kinetic\ Energy/mc^2$ for Einstein (blue curve), Newton (black dotted line) and for the data by Lund and Uggerhøj; graph borrowed from [10]

Note that in Figure 5, the plot is v_1/c versus $Kinetic\ Energy/mc^2$ whereas the plot in Figure 4 is $Kinetic\ Energy/mc^2$ versus $(v_1/c)^2$. At high speeds with v_1/c greater than 0.941 (or $(v_1/c)^2$ greater than 0.8855), the measured kinetic energies are again significantly larger than those provided by Einstein's formula, as found in Bertozzi's data.[7] For the two data points with the two highest energies, Einstein's formula underestimates the measured kinetic energies by 0.424 and 1.039 times of the electron's rest energy. The trend indicated in the data by Lund and Uggerhøj and the data by Bertozzi is that *the kinetic energy deficit in Einstein's formula becomes evident at high speed, and the largest deficit is found at the highest measured speed in each experiment*, which new formula predicts.

Because of the weightiness of the matter, even though there are the five data points (three from Bertozzi and two from Lund and Uggerhøj) confirming the clear deficit in Einstein's energy formula for high speed, more experimental data need to be collected in order to further confirm the underestimation in Einstein's formula as predicted by the new formula. At this point, having compared Einstein's formula with the existing observed data, it will be instructive to see if the new formula will fit the existing observed data better than Einstein's formula. We will take a least squares approach to assess and compare the fit of the new formula to the existing observed data with the fit of Einstein's formula to the data.

We compare the two fits of the two formulae to Bertozzi's data and will discuss this in relation to those of Lund and Uggerhøj later. Since the data point B4 in Bertozzi's data gives an energy deficit in Einstein's formula of 3.6 times of the electron's rest/base energy, including this data point in the comparison may significantly disadvantage Einstein's formula. Hence, apart from using all four points in Bertozzi's data for the comparison of the two fits, we will also use the first three points in Bertozzi's data (B1, B2 and B3) for such a comparison.

For the first three points in Bertozzi's data (B1, B2 and B3), using $(v_1/c)^2$ as the independent variable, the sum of the squares of the differences between the normalised kinetic energy in Bertozzi's data and the normalised kinetic energy given by Einstein's

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[7] The data for the measured kinetic energy and speed of the electrons are kindly provided by Professor Uggerhøj. These data are used to calculate the underestimation of the kinetic energy by Einstein's formula.

formula, i.e., the sum of the squares of the three $\triangle(Kinetic\,Energy/mc^2)$ for B1, B2 and B3 in Table 1, is 0.4247.

For the same three points, similarly using $(v_1/c)^2$ as the independent variable, the sum of the squares of the differences between the normalised energy in Bertozzi's data and the normalised energy given by the new formula depends on the parameter, m_i^2 / m^2, as can be seen in (§14). One has to find the best parameter value to minimise the sum of the squares. The best parameter value for m_i^2 / m^2 in this case is 0.275 (hence this value is used in Figure 3). With this parameter value, the sum of the squares of $\triangle(Kinetic\,Energy/mc^2)$ for the new formula is 0.1385, which is significantly smaller than 0.4247, the summed squares for Einstein's formula. This is to be expected since Einstein's formula corresponds to zero value for m_i^2 / m^2 and the least squares approach yields the least summed squares with the parameter value of 0.275 rather than zero.

We repeat the above procedure by using all four points in Bertozzi's data, again using the best paramter for m_i^2 / m^2 (1.0725) to minimise the summed squares for the new formula. The results from the two procedures are summarised in the following table.

Data Points	**Formula**	m_i^2 / m^2	**Sum of squares of △K.E. / *mc*²**	**△K.E. / *mc*² at B4**
B1 to B3	Einstein	0	0.4247	
B1 to B3	New	0.275	0.1385	
B1 to B4	Einstein	0	13.4175	8.8063-5.2 =3.6063
B1 to B4	New	1.0725	2.7561	8.8063-7.868 =0.9383

Table 2: Sum of squares of △K.E. / mc^2 for Einstein formula and the new formula, based on Bertozzi's data; and the kinetic energy deficit compared to the data at B4

As expected, Einstein's formula does better with the first three data points, B1 to B3, than with all four data points (because Einstein's formula greatly underestimates the kinetic energy for the fourth data point, B4). For both the cases of three and four data points, the new formula predicts the kinetic energy better than Einstein's formula when the criterion of least squares is applied in the comparison. In fact, for the case of four data points where the data point with the highest kinetic energy is included, (i) generally in terms of the least squares criterion the new formula (with summed squares=2.7561) predicts the kinetic energy far better than Einstein's formula (with summed squares=13.4175), and (ii) specifically in terms of the particular normalised energy deficit at B4, the new formula (with 0.9383) also does far better than Einstein's formula (with 3.6063).

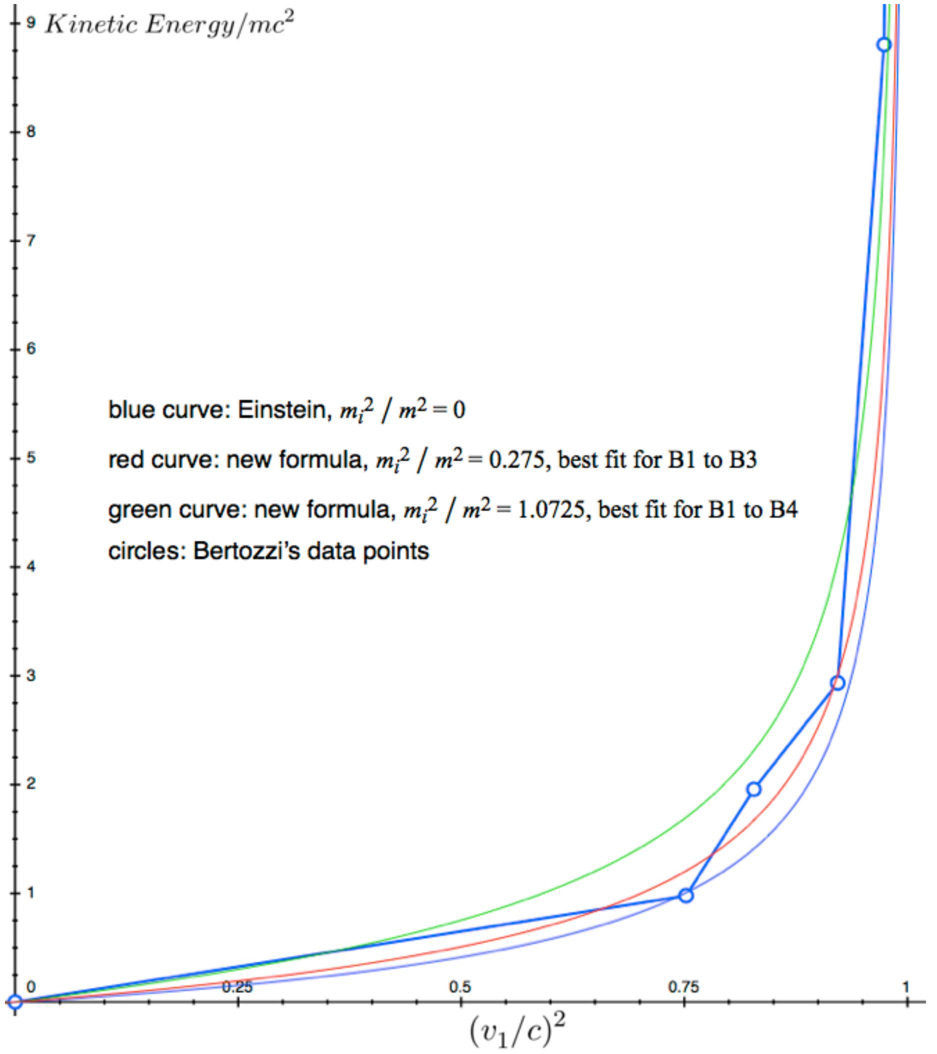

Figure 6: K.E. / mc² vs $(v_1/c)^2$ for for Einstein's formula ($m_i^2 / m^2 = 0$), the new formula ($m_i^2 / m^2 = 0.275$ and 1.0725), and Bertozzi's four data points

Figure 6 shows the new formula's two least squares fits to Bertozzi's data (using his three or four data points). Again, for B2 to B4 with higher kinetic energy than B1, *these data points fall on the left side of the curve for Einstein's formula and the largest energy deficit in that formula with reference to Bertozzi's data occurs at the point with the hightest energy*, B4. This trend is echoed by the two highest energy data points from Lund and Uggerhøj [10] in Figure 5.

One could repeat the application of the above procedure of comparison to the data by Lund and Uggerhøj, i.e., comparing the fits of Einstein's formula and the new formula to their data. But we need to be careful here because of the paucity of data with high energy in their data compared to Bertozzi's data. In Figure 5 for the data from Lund and Uggerhøj, the kinetic energy deficit in Einstein's formula becomes evident at high speeds with v_1/c greater than 0.941 ($(v_1/c)^2$ greater than 0.8855 or normalised kinetic energy greater than 1.955), while Bertozzi's data suggest the kinetic energy deficit in Einstein's formula becomes evident at high speeds with $(v_1/c)^2$ greater than 0.828 (or normalised kinetic energy greater than 1.957), which is also suggested by Figure 3. From these, it is reasonable to suggest that *the point with normalised kinetic energy equal to 2 is a critical point, around and beyond which the kinetic energy deficit in Einstein's formula becomes evident*. If the two points in the data by Lund and Uggerhøj with the highest speed and kinetic energy (which are close to or beyond the critical normalised kinetic energy of 2) are excluded from the dataset, the best least squares fit to this truncated dataset using the new formula will involve the parameter, m_i^2 / m^2, having nearly zero value, i.e., the best fit curve using the new formula will be close to Einstein's curve. That is, the problem of kinetic energy deficit in Einstein's formula will not be evident from the truncated dataset. To unravel that problem of kinetic energy deficit, one needs more data points with normalised kinetic energy near to or greater than 2 (see Figure 5). But since there are only two such data points from the dataset by Lund and Uggerhøj, in order to unravel the kinetic energy deficit problem with this limited number of data points with such high energy, one should include these two data points with normalised kinetic energy greater than 2 and include a comparable number of data point with normalised kinetic energy less than 2. That is, we are trying to give equal weight to the section of the curve with normalised kinetic energy less than 2 (where the kinetic energy deficit in Einstein's formula is not so evident) and the section of the curve with normalised kinetic energy greater than 2 (where the kinetic energy deficit in Einstein's formula is evident). Here we use (i) the three data points with normalised kinetic energy less than 2 and closest to it (these points lie close to Einstein's curve), and (ii) the two data points with normalised kinetic energy greater than 2 (2.9354 and 3.914); see Figure 5. With these five data points, using the new formula the least squares fit requires m_i^2 / m^2 to have the value of 0.26 and the minimum summed squared for $\triangle(Kinetic\, Energy/mc^2)$ is 0.6462.[8] If we set the value of m_i^2 / m^2 to zero in the new formula, then the new formula is equivalent to Einstein's formula and the summed squared for $\triangle(Kinetic\, Energy/mc^2)$ is 1.2606 which is almost twice of 0.6462, the value for the best fit curve for the new formula. This shows again that the new formula performs better than Einstein's formula in terms of the least squares criterion when enough data points with sufficiently high speed (or with normalised kinetic energy near to or greater than 2) are included in the dataset.

5.0 Discussion

We can collect the following three summary points from our foregoing discussion. Firstly, it has been seen in both the dataset by Bertozzi and the dataset by Lund and Uggerhøj that an electron with normalised kinetic energy near to or greater than 2 is not well represented by Einstein's formula, i.e., its kinetic energy exceeds the kinetic energy predicted by Einstein's formula for the observed speed. Secondly, both datasets show that the highest kinetic deficit in Einstein's formula occurs at the data point with the highest kinetic energy or highest speed. These two summary points are predicted by the new formula. Thirdly, it has also been shown that according to the criterion of least squares the new formula performs better than Einstein's formula in representing the relationship between kinetic energy and speed when data points with normalised kinetic energy near to or greater than 2 are included. It is noted that when the normalised kinetic energy is plotted against the normalised speed (or its square), the plot readily yields the kinetic energy deficit in Einstein's formula (relative to the observed energy) for a given speed (see Figures 4 and 6). This has made the energy deficit or

[8] The curve for this parameter value of 0.26 is virtually indistinguishable from the red curve for 0.275 in Figure 6.

anomaly in Einstein's formula evident. The energy deficit/anomaly in that formula will be more hidden if speed is plotted again energy, as is often the case (see Figure 5 from Lund and Uggerhø [10] and the original plot in Bertozzi's paper [9]); this is because in that case one tends to read off the discrepancy in speed (not energy) between the speed given by the formula and the observed speed for a given energy, and this discrepancy in speed at high energy will invariably be small since both speeds will be close to c. One will then conclude that there is little discrepancy in speed between observation and Einstein's formula for a given energy. This conclusion is correct. However, the clear and often large kinetic energy deficit/anomaly in Einstein's formula (compared to the observed kinetic energy) for a given high speed still stands, as revealed by the plot with kinetic energy versus speed (Figures 4 and 6). The fact that Lund and Uggerhø [10] and Bertozzi [9] plotted speed against energy may account for their lack of discussion on the kinetic energy deficit/anomaly in Einstein's formula for high speed.

The fact that, according to the criterion of least squares (of the kinetic energy discrepancy between the energy given by a formula and the observed energy for a given speed), the new formula performs better than Einstein's formula does not necessarily show beyond reasonable doubt that the new formula is better than Einstein's formula. This is because we have only utilised five data points with normalised kinetic energy greater than 2 (three from Bertozzi and two from Lund and Uggerhøj). Further comparison between the two formulae ought to be made with more data points with normalised kinetic energy greater than the critical threshold value of 2.[9] It is possible that such data already exist, e.g., in the CERN massive dataset. What are required from these datasets for such comparisons are the accurate values of speed (not momentum) and the corresponding values of normalised kinetic energy. In terms of the relationship between translational momentum and energy, (§1) (for Einstein's formula) and (§7) (for the new formula) give the same form of relationship. What makes (§7) distinctive from (§1) is their different definitions of the translational momentum and these different definitions are expressed in terms of speed in different ways (see (§2) and (§5)). Hence, speed (not momentum) and kinetic energy are required to unravel the distinction between Einstein's formula (§1) and the new formula (§7) and so make possible their comparison. It needs to be noted that to evaluate the performance of either of

[9] This threshold value may vary slightly for different particles; see Figure 3 with different parameters for the inherent mass corresponding to different particles.

the two theoretical formulae using the criterion of least summed squares as performed in the last section, the speed of the particle has to be measured to high accuracy. This is because a small error in the measured speed can add a large value to the difference between the measured kinetic energy and the kinetic energy predicted by the theoretical formula (the one from Einstein or the new one). This is because at high speed the kinetic energy predicted by either theoretical formula is highly sensitive to the value of the particle speed input into the formula; see Figure 2.

Before more comparisons between the performances of the two formulae are made, one cannot come to a firm conclusion about the validity of the new formula. But one can consider the possible implications *if* indeed further comparisons made with more observed data confirm that the new formula does perform better than Einstein's formula. Such a prospective consideration about the implications is not premature here because such consideration may encourage us to make those further comparisons using observed data with a greater sense of urgency.

The first implication to be considered is for our understanding of dark matter. Admittedly, the kinetic energy deficit in Einstein's formula becomes more evident only for $(v_1/c)^2$ greater than 0.8 or 0.9 according to the analysis above for electrons, i.e., the deficit in Einstein's formula will not be that significant for the usual slower speeds. Here we need to consider the meaning of the parameter, m_i^2 / m^2, for the new formula. If *m,* the effective or rest mass, is invariant for a particular kind of particle, and if m_i, the 'inherent mass' of the particle, is also invariant, then the parameter of m_i^2 / m^2 is invariant for that kind of particle. And different kinds of particle can have different values for m_i^2 / m^2. Their values can be ascertained by the best fit (least squares) procedure as carried out in the the previous section. It is conceivable that for some kinds of particle m_i^2 / m^2 can be larger than 1 so that, according to Figure 2 and Figure 3, the kinetic energy deficit in Einstein's formula for these particles will be larger than those for electrons and the deficit could become more evident at $(v_1/c)^2$ much less than 0.8 or 0.9. At present, it is not known what m_i^2 / m^2 values different kinds of particle will have and so one cannot know with confidence to what extent Einstein's formula underestimates the kinetic energy of different kinds of particles. Nevertheless, because of the large magnitude of the dark matter, it may be unlikely that dark matter can be substantially explained by the underestimation of the kinetic energy of particles in Einstein's formula. An additional and a different explanation may well be necessary. And this additional explanation might have something to do with annihilation of a pair of particle and anti-particle.

For zero $\underset{\sim}{v}_s$, i.e., zero surfing velocity and therefore zero surfing momentum (p_s), the effective/rest mass, *m*, will be zero even though its inherent mass, m_i , is not zero. This notion of a 'massless' entity could be applied to the joint particle and anti-particle pair after annihilation in the following manner. Before annihilation, the particle had effective/rest mass, *m,* which was generated from the inherent mass, m_i , by the surfing motion on the *S* surface ($m = p_s/c$), and the rest/base energy is given by the surfing energy, $p_s c = mc^2$; and the anti-particle also had the same effective/rest mass as the particle and the same rest/base energy as the particle by the same surfing mechanism. The particle and anti-particle had opposite spin and opposite charge (if any). As the particle and anti-particle came together, i.e., at the so-called annihilation, the surfing energy of the particle and the surfing energy of the anti-particle are released in two photons, each with energy of $p_s c = mc^2$. After 'annihilation', the joint particle and anti-particle entity has zero surfing momentum and therefore zero surfing energy and zero rest/effective mass; the joint entity also has zero spin (because of no surfing motion) and zero charge. This joint entity is not discernible as far as its effective/rest mass, spin and charge are concerned. However, even though its effective/rest mass is zero, such a 'massless' joint entity can have non-zero translational momentum because of its non-zero inherent mass if its translational speed is non-zero (see (§5)); and its non-zero energy (kinetic) is given by

$$E_N = p_{1N} c$$

according to (§7) where the inherent mass of the joint entity, appearing in p_{1N}, is understood to be twice m_i . (Note that if we apply Einstein's formula to this joint entity, since the rest mass is zero, the translational momentum is zero according to (§2); the kinetic energy and the total energy, *E*, will also be zero according to (§1).) This notion of non-zero energy for a joint 'massless', spin-less and charge-less entity formed from 'annihilation' could, at least partially, account for the elusive dark matter and the energy thereof, i.e., there could be many such joint 'massless' entities spread out in the vast space in a galaxy with non-zero kinetic energies which are undetectable in terms of their spin and charge but their presence is manifested through their kinetic energies which add to the total energy of the galaxy. And when such a hidden joint entity receives sufficient energy, i.e., greater than twice the base/ rest energy of the particle, the particle and anti-particle in the joint entity will be separated, or 'created', each with its own surfing momentum (p_s) and therefore its own effective/rest mass ($m = p_s/c$) and its own surfing energy ($p_s c = mc^2$), its own spin (because of the non-zero surfing motion) but of opposite signs in order to conserve the total angular momentum,[10] and its own charge (if any) of opposite signs. Effectively, this particle and anti-particle 'creation' mechanism is the reverse of the 'annihilation' mechanism. This manner of envisaging particle 'annihilation' and 'creation' has the advantage that it does not involve the mysterious disappearance of matter at 'annihilation' and the mysterious appearance of matter at 'creation'. These matters, or more precisely these particles and anti-particles, are always there but they either exist individually with non-zero effective/rest mass or exist in a joint particle and anti-particle form with zero effective/rest mass. In either case, they can contribute energy to the total energy of the galaxy.

6.0 Conclusion

One must remember that the above analysis concerning dark matter is a conjecture based on the assumption that the new formula is preferable to Einstein's formula. But this conjecture has been given here to illustrate the kind of possible implications for physics the new formula can have so that ascertaining whether indeed the new formula is preferable can be deemed as an important task. If indeed the new formula is further demonstrated to be preferable by observed data, this will also add credibility to the relativistic model (and the associated non-relativistic model) for quantum particles (both models utilise the notion of surfing motion) which is adopted in this paper and was suggested by the author in previous papers, e.g., [8]. This model was briefly explained in section 2 of this paper but it has been

[10] If the particle has zero spin, i.e., $\underset{\sim}{v}_2$ is zero, it can still have surfing motion and thus non-zero surfing momentum because $\underset{\sim}{v}_s \equiv \underset{\sim}{v}_2 + \underset{\sim}{v}_3$.

shown in [8] that the two models are consistent with or give explanation to a significant number of observed phenomena in quantum mechanics (see section 10.2 on Conclusion in [8]):

1. non-determinacy
2. de Broglie relation between momentum and wavelength
3. Planck-Einstein relation between energy and frequency
4. Born's rule
5. spin
6. ontological particle nature of *both* particles and photons
7. the observed wavy functional behaviour of *both* particles and photons
8. the interference pattern observed in the two-slit experiment for slowing moving particles and the interference pattern of the same experiment for relativistic particles and photons
9. measurement which destroys the interference pattern of the two-slit experiment
10. tunnelling
11. rest energy.

The new formula for energy is an important outcome of the relativistic model in [8]. Validation of the new formula by further observation data, if it turns out to be the case, will make the relativistic model and its associated non-relativistic model more credible.

Hestenes [1,5], Lepadatu [6] and Rockenbauer [7] associated the rest energy and rest mass of a particle with the spinning motion of the particle. Their approaches require the particle to move at the speed of light. This paper echoes their approaches but takes quite a different route where (i) the particle is not required to move at the speed of light, and (ii) the rest energy and rest mass of a particle is associated with the surfing velocity of the particle on the phase (*S*) surface, $\underset{\sim}{v}_s \equiv \underset{\sim}{v}_2 + \underset{\sim}{v}_3$. $\underset{\sim}{v}_2$ can be called a spin velocity component but there is also the other component they did not consider, $\underset{\sim}{v}_3$, which is perpendicular to $\underset{\sim}{v}_2$. Furthermore, this paper has worked out systematically the details of how the surfing momentum is related to the rest/effective mass ($m = p_s/c$) and the rest/base energy ($p_s c = mc^2$) by invoking the notion of inherent mass, m_i, which is absent in their models. By suggesting an alternative definition of the energy of a particle which is partially supported by experimental data, it gives us possible (or plausible) insights into the origin or generation of rest mass, the nature of so-called massless particles, and the mechanism for 'annihilation' and 'creation' of a pair of particle and anti-particle. These may have implications for our understanding of dark matter since the 'massless' entity, resulted from the joining of the particle and anti-particle together, is supposed to be annihilated out of existence, but this paper suggests that the 'massless' joint entity can have non-zero kinetic energy while its net spin and net charge are zero.

Finally, at the end of his paper, Rockenbauer hypothesised, 'In the fusion or fission processes of atomic nuclei, the vast energy of escaping irradiation is supplied by the partial loss of the spinning kinetic energy of nucleons.' This paper almost agrees with his hypothesis but will replace the term 'spinning kinetic energy' with 'surfing kinetic energy'. This could have implications for our understanding of strong nuclear force.

(Word count: 7893)

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