Comment on "Describing Weyl neutrinos by a set of Maxwell-like equations" by S. Bruce (*)

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Summary. — Results of the work of S. Bruce (*Nuovo Cimento B*, **110** (1995) 115) are compared with those of the recent papers of D. V. Ahluwalia and myself, devoted to describing neutral particles of spin $j = 1/2$ and $j = 1$.

PACS 03.50 – Classical field theory.
PACS 03.65.Pm – Relativistic wave equations.
PACS 11.30.Cp – Lorentz and Poincaré invariance.
PACS 11.30.Er – Charge conjugation, parity, time reversal, and other discrete symmetries.

The main result of ref. [1] is the proof of the possibility of deriving the generalized Maxwell's equations (eqs. (24) of the cited paper) from a set of the $j = 1/2$ equations for Weyl spinors. Another important point discussed there is the connection between parity-even and parity-odd parts of the CP conjugate states. The consideration is restricted by the massless case.

On the other hand, in recent papers of Ahluwalia and Dvoeglazov [2] on the basis of ideas of Majorana [3] and McLennan and Case [4] the construct for self/anti-self charge conjugate states has been presented. It permits one to take into account possible effects of neutrino mass and to explain origins of the question of "missing" right-handed neutrino [5]. Type-II self/anti-self charge conjugate spinors are defined in the momentum representation from the beginning, in the following way (ref. [2*a*], eq. (6)):

$$\lambda(p^\mu) \equiv \begin{pmatrix} (\zeta_\lambda \Theta_{[j]})\, \phi_L^*(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix}, \qquad \varrho(p^\mu) \equiv \begin{pmatrix} \phi_R(p^\mu) \\ (\zeta_\varrho \Theta_{[j]})^* \phi_R^*(p^\mu) \end{pmatrix}. \tag{1}$$

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ζ_λ and ζ_ϱ are the phase factors that are fixed by the conditions of self/anti-self conjugacy, $\Theta_{[j]}$ is the Wigner time-reversal operator for spin j. They satisfy the equations ([1])

$$i\gamma^\mu \partial_\mu \lambda^{\mathrm{S}}(x) - m\varrho^{\mathrm{A}}(x) = 0\,, \tag{2a}$$

$$i\gamma^\mu \partial_\mu \varrho^{\mathrm{A}}(x) - m\lambda^{\mathrm{S}}(x) = 0\,, \tag{2b}$$

$$i\gamma^\mu \partial_\mu \lambda^{\mathrm{A}}(x) + m\varrho^{\mathrm{S}}(x) = 0\,, \tag{2c}$$

$$i\gamma^\mu \partial_\mu \varrho^{\mathrm{S}}(x) + m\lambda^{\mathrm{A}}(x) = 0\,. \tag{2d}$$

It was shown there (cf. ref. [2*b*], eqs. (22) or [2*c*], eqs. (67)-(70)) that type-II spinors are connected with the Dirac spinors in the Weyl representation $u_\sigma(p^\mu)$ and $v_\sigma(p^\mu)$ as follows:

$$\lambda^{\mathrm{S}}_{\uparrow}(p^\mu) = \frac{1}{2}\left(u_{+1/2}(p^\mu) + iu_{-1/2}(p^\mu) - v_{+1/2}(p^\mu) + iv_{-1/2}(p^\mu)\right)\,, \tag{3a}$$

$$\lambda^{\mathrm{S}}_{\downarrow}(p^\mu) = \frac{1}{2}\left(-iu_{+1/2}(p^\mu) + u_{-1/2}(p^\mu) - iv_{+1/2}(p^\mu) - v_{-1/2}(p^\mu)\right)\,, \tag{3b}$$

$$\lambda^{\mathrm{A}}_{\uparrow}(p^\mu) = \frac{1}{2}\left(u_{+1/2}(p^\mu) - iu_{-1/2}(p^\mu) - v_{+1/2}(p^\mu) - iv_{-1/2}(p^\mu)\right)\,, \tag{3c}$$

$$\lambda^{\mathrm{A}}_{\downarrow}(p^\mu) = \frac{1}{2}\left(iu_{+1/2}(p^\mu) + u_{-1/2}(p^\mu) + iv_{+1/2}(p^\mu) - v_{-1/2}(p^\mu)\right)\,. \tag{3d}$$

and (ref. [2*a*], eqs. (48))

$$\varrho^{\mathrm{S}}_{\uparrow}(p^\mu) = -\,i\lambda^{\mathrm{A}}_{\downarrow}(p^\mu)\,, \qquad \varrho^{\mathrm{A}}_{\uparrow}(p^\mu) = +\,i\lambda^{\mathrm{S}}_{\downarrow}(p^\mu)\,, \tag{4a}$$

$$\varrho^{\mathrm{S}}_{\downarrow}(p^\mu) = +\,i\lambda^{\mathrm{A}}_{\uparrow}(p^\mu)\,, \qquad \varrho^{\mathrm{A}}_{\downarrow}(p^\mu) = -\,i\lambda^{\mathrm{S}}_{\uparrow}(p^\mu)\,. \tag{4b}$$

We assumed that $\phi^{+1/2}_{\mathrm{L,R}}(\mathring{p}^\mu) = \mathrm{column}\ (1\ \ 0)$, $\phi^{-1/2}_{\mathrm{L,R}}(\mathring{p}^\mu) = \mathrm{column}\ (0\ \ 1)$; in the opposite case we have to include additional phase factors in the mass terms of eqs. (2*a*)-(2*d*). They can be fixed if the theory is implied invariant with respect to the intrinsic parity.

Using identities (4*a*), (4*b*) and rewriting eqs. (2*a*)-(2*d*) in the momentum representation with taking into account the chiral helicity quantum number [2] we are able to obtain the following equations in the two-component form (phase factors are restored):

$$[p^0 + \boldsymbol{\sigma}\cdot\mathbf{p}]\,\phi^{\uparrow}_{\mathrm{L}}(p^\mu) - me^{+i\chi}\,\Theta\phi^{\downarrow}_{\mathrm{L}}{}^{*}(p^\mu) = 0\,, \tag{5a}$$

$$[p^0 - \boldsymbol{\sigma}\cdot\mathbf{p}]\,\Theta\phi^{\uparrow}_{\mathrm{L}}{}^{*}(p^\mu) + me^{-i\chi}\,\phi^{\downarrow}_{\mathrm{L}}(p^\mu) = 0\,, \tag{5b}$$

$$[p^0 + \boldsymbol{\sigma}\cdot\mathbf{p}]\,\phi^{\downarrow}_{\mathrm{L}}(p^\mu) + me^{+i\chi}\,\Theta\phi^{\uparrow}_{\mathrm{L}}{}^{*}(p^\mu) = 0\,, \tag{5c}$$

$$[p^0 - \boldsymbol{\sigma}\cdot\mathbf{p}]\,\Theta\phi^{\downarrow}_{\mathrm{L}}{}^{*}(p^\mu) - me^{-i\chi}\,\phi^{\uparrow}_{\mathrm{L}}(p^\mu) = 0\,, \tag{5d}$$

([1]) As we got knowing recently, this set of equations has been proposed in ref. [6] for the first time.

which answer for the McLenann-Case-Ahluwalia construct. A remarkable feature of this set is that it is valid both for positive- and negative-energy solutions of eqs. (2*a*)-(2*d*). The corresponding equations for $\phi_R(p^\mu)$ and $\Theta\phi_R^*(p^\mu)$ spinors follow after substitution $\mathbf{p}\to-\mathbf{p}$ in the matrix structures of the equations (*not* in the spinors!). The phase factors $\chi_{R,L}\equiv\vartheta_1^{R,L}+\vartheta_2^{R,L}$ are defined by explicit forms of the 2-spinors of different helicities (2) and can be regarded at this moment as arbitrary.

Considering properties of 4-spinors with respect to the $\mathbf{p}\to-\mathbf{p}$ after Bruce we state (3)

$$(6)\qquad \xi^L_{\rm even}=\phi_L^\downarrow+\Theta\phi_L^{\uparrow *}\equiv\begin{pmatrix}B^3+iB^0\\ B^1+iB^2\end{pmatrix},\qquad \xi^L_{\rm odd}=\phi_L^\downarrow-\Theta\phi_L^{\uparrow *}\equiv\begin{pmatrix}-E^0+iE^3\\ -E^2+iE^1\end{pmatrix}.$$

The opposite helicity parts are connected by the Wigner time-reversal operator:

$$(7)\qquad \begin{cases}\phi_L^\uparrow+\Theta\phi_L^{\downarrow *}\equiv\Theta\mathcal{K}\xi^L_{\rm odd}=\begin{pmatrix}E^2+iE^1\\ -E^0-iE^3\end{pmatrix},\\[2ex] \phi_L^\uparrow-\Theta\phi_L^{\downarrow *}\equiv-\Theta\mathcal{K}\xi^L_{\rm even}=\begin{pmatrix}B^1-iB^2\\ -B^3+iB^0\end{pmatrix}.\end{cases}$$

Adding and subtracting relevant equations of the set (5*a*)-(5*d*) we obtain

$$(8a)\qquad \begin{pmatrix}p^0&0\\0&p^0\end{pmatrix}\begin{pmatrix}E^2+iE^1\\ -E^0-iE^3\end{pmatrix}+\begin{pmatrix}p^3&p^1-ip^2\\ p^1+ip^2&-p^3\end{pmatrix}\begin{pmatrix}B^1-iB^2\\ -B^3+iB^0\end{pmatrix}-$$
$$-m\cos\chi\begin{pmatrix}E^2+iE^1\\ -E^0-iE^3\end{pmatrix}+im\sin\chi\begin{pmatrix}B^1-iB^2\\ -B^3+iB^0\end{pmatrix}=0\,,$$

$$(8b)\qquad \begin{pmatrix}p^0&0\\0&p^0\end{pmatrix}\begin{pmatrix}B^1-iB^2\\ -B^3+iB^0\end{pmatrix}+\begin{pmatrix}p^3&p^1-ip^2\\ p^1+ip^2&-p^3\end{pmatrix}\begin{pmatrix}E^2+iE^1\\ -E^0-iE^3\end{pmatrix}+$$
$$+m\cos\chi\begin{pmatrix}B^1-iB^2\\ -B^3+iB^0\end{pmatrix}-im\sin\chi\begin{pmatrix}E^2+iE^1\\ -E^0-iE^3\end{pmatrix}=0\,,$$

$$(8c)\qquad \begin{pmatrix}p^0&0\\0&p^0\end{pmatrix}\begin{pmatrix}B^3+iB^0\\ B^1+iB^2\end{pmatrix}+\begin{pmatrix}p^3&p^1-ip^2\\ p^1+ip^2&-p^3\end{pmatrix}\begin{pmatrix}-E^0+iE^3\\ -E^2+iE^1\end{pmatrix}+$$
$$+m\cos\chi\begin{pmatrix}B^3+iB^0\\ B^1+iB^2\end{pmatrix}-im\sin\chi\begin{pmatrix}-E^0+iE^3\\ -E^2+iE^1\end{pmatrix}=0\,,$$

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(2) Of course, different choices of $\chi_{R,L}$ will influence eqs. (4*a*), (4*b*).

(3) We prefer to use the conventional notation $\mathbf{M}\to\mathbf{E}$, the polar vector, and $\mathbf{N}\to\mathbf{B}$, the axial vector. We still leave room for different interpretations of these vectors in relevant physical cases.

$$(8d)\quad \begin{pmatrix} p^0 & 0\\ 0 & p^0\end{pmatrix}\begin{pmatrix} -E^0+iE^3\\ -E^2+iE^1\end{pmatrix}+\begin{pmatrix} p^3 & p^1-ip^2\\ p^1+ip^2 & -p^3\end{pmatrix}\begin{pmatrix} B^3+iB^0\\ B^1+iB^2\end{pmatrix}-$$
$$-m\cos\chi\begin{pmatrix} -E^0+iE^3\\ -E^2+iE^1\end{pmatrix}+im\sin\chi\begin{pmatrix} B^3+iB^0\\ B^1+iB^2\end{pmatrix}=0\ .$$

They are recast into the vector form

$$(9a)\quad \mathbf{p}\times\mathbf{E}-p^0\mathbf{B}+\mathbf{p}E^0-m\mathbf{B}\cos\chi-m\mathbf{E}\sin\chi=0\ ,$$

$$(9b)\quad \mathbf{p}\times\mathbf{B}+p^0\mathbf{E}+\mathbf{p}B^0-m\mathbf{E}\cos\chi+m\mathbf{B}\sin\chi=0\ ,$$

$$(9c)\quad p^0E^0-(\mathbf{p}\cdot\mathbf{B})-mE^0\cos\chi+mB^0\sin\chi=0\ ,$$

$$(9d)\quad p^0B^0+(\mathbf{p}\cdot\mathbf{E})+mB^0\cos\chi+mE^0\sin\chi=0\ .$$

For parity conservation of these vector equations we should assume that E_0 would be a pseudoscalar and B_0 would be a scalar, furthermore, $\chi=0$ or π. In a matrix form with the Majorana-Oppenheimer matrices

$$(10a)\quad \alpha^0=\mathbf{1}_{4\times 4},\qquad \alpha^1=\begin{pmatrix} 0 & -1 & 0 & 0\\ -1 & 0 & 0 & 0\\ 0 & 0 & 0 & -i\\ 0 & 0 & i & 0\end{pmatrix},$$

$$(10b)\quad \alpha^2=\begin{pmatrix} 0 & 0 & -1 & 0\\ 0 & 0 & 0 & i\\ -1 & 0 & 0 & 0\\ 0 & -i & 0 & 0\end{pmatrix},\qquad \alpha^3=\begin{pmatrix} 0 & 0 & 0 & -1\\ 0 & 0 & -i & 0\\ 0 & i & 0 & 0\\ -1 & 0 & 0 & 0\end{pmatrix},$$

we can obtain the equations ($\overline{\alpha}^0\equiv\alpha^0$, $\overline{\alpha}^i\equiv-\alpha^i$)

$$(11a)\quad \alpha^\mu p_\mu\Psi_2(p^\mu)-me^{-i\chi}\Psi_1(p^\mu)=0\ ,$$

$$(11b)\quad \overline{\alpha}^\mu p_\mu\Psi_1(p^\mu)-me^{+i\chi}\Psi_2(p^\mu)=0\ ,$$

satisfied by the field functions

$$(12a)\quad \Psi_1(p^\mu)=-\mathscr{C}\Psi_2^*(p^\mu)=\begin{pmatrix} -i(E^0-iB^0)\\ E^1-iB^1\\ E^2-iB^2\\ E^3-iB^3\end{pmatrix},\quad \Psi_2(p^\mu)=-\mathscr{C}\Psi_1^*(p^\mu)=\begin{pmatrix} -i(E^0+iB^0)\\ E^1+iB^1\\ E^2+iB^2\\ E^3+iB^3\end{pmatrix},$$

where

$$\mathscr{C} = \mathscr{C}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \mathscr{C}\alpha^\mu \mathscr{C}^{-1} = \overline{\alpha}^{\mu *} \tag{13}$$

in accordance with the definition of Dowker [7] in the orthogonal basis. Some remarks have already been done that these equations can be written in the same form for both positive- and negative-frequency solutions. If we choose $\Psi_2(p^\mu)$ as presenting these parts of the field operator and $\chi = \pi$, then we have in the coordinate representation:

$$i\,\wp_{u,v}\alpha^\mu \partial_\mu \Psi(x^\mu) - m\,\Psi^c(x^\mu) = 0\,, \tag{14a}$$

$$i\,\wp_{u,v}\overline{\alpha}^\mu \partial_\mu \Psi^c(x^\mu) - m\Psi(x^\mu) = 0\,, \tag{14b}$$

with $\wp_{u,v} = \pm 1$ depending on what solutions, of positive or negative frequency, are considered (cf. [8]).

Next, we would like to mention that similar formulations (but without a mass term) were met in literature [9-12]. Probably, applying them was caused by some shortcomings of eq. (1) of the paper [1], which the author of the commented work paid attention to. To his critical remarks we can add that it contains the acausal solution with $E = 0$, ref. [9, 10, 13]. Acausal solutions of similar nature appear for any spin (not only for spin-1 equations), ref. [13]. While Oppenheimer proposed a physical interpretation of this solution as connected with electrostatic solution and recently another solution, the $B(3)$ longitudinal field, to the Maxwell's equations was extensively discussed [14], the problem did not yet find an adequate consideration. In the mean time, the equations for spin-1 massless bosons, presented by Majorana, Oppenheimer, Giantetto, and the ones of this paper for massive spin-1 case, are free of any acausalities; they are of the first order in time derivatives and represent the Lorentz-invariant theory ([4]). As a price we have additional displacement current and a possible mass term.

Other equations which can be considered as suitable candidates for describing spin-1 bosons are the second-order Weinberg equations [15, 13, 16]; they have only causal solutions $E = \pm p$ in the massless limit. Moreover, their massless limit also can be reduced [16] to eqs. (24) of the commented paper.

Finally, I would like to note that presented ideas deserve further rigorous elaboration, since we are still far from understanding the nature of electron, photon and neutrino. Their specific features seem not to lie in some specific representation of the Poincaré group but in the structure of our space-time. Thus, the equations for the fields in the $(1, 0) \oplus (0, 0)$ and the $(0, 1) \oplus (0, 0)$ representations, given above, could provide additional information for our goals.

([4]) The question of the relativistic invariance of the equations (14*a*), (14*b*) is a tune point. A separate paper will discuss the relativistic invariance of new equations. But one should note here that providing new frameworks we are not going to dispute results of the Dowker's consideration ((7*a*), p. 183).

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