A question about the package of $L$-functions of the symmetric $n$-th powers of a given automorphic form

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Abstract

The recent work of Clozel, Harris, Shepherd-Baron, and Taylor on the Sato-Tate conjecture makes it natural to ask questions such as hinted at in the title. Some ideas of Sarnak and Sarnak-Rubenstein directly connects such questions with the historic hunch of Birch and Swinnerton-Dyer that if an elliptic curve $E$ over $\mathbb{Q}$ has infinitely many rational points, then for varying prime numbers $p$, the estimate $p + 1$ will be preponderantly an under-count for its number of rational points when reduced mod $p$, while if $E$ has only finitely many rational points it will be an over-count. William Stein, using SAGE, has begun the project of amassing data relevant to this. I will discuss all this in my lecture.