I will describe a result obtained (dis)jointly with L. Dieulefait and J. Jiménez of Barcelona: given a positive integer $t$ and a bound $B > 0$, we find distinct prime numbers $p_1, \ldots, p_t$ and a weight-2 newform on $\Gamma_0(p_1 \cdots p_t)$ whose field of coefficients has degree bigger than $B$.

The case $t = 1$ was discussed by Mazur in his Eisenstein ideal article, and the case $t > 2$ can be handled by an amazing Barcelona argument that uses Chen’s weak version of the Goldbach conjecture. One can prove the desired result when $t = 2$ by studying the new Eisenstein primes in the Hecke algebra attached to the space of forms of level $p_1p_2$. It is undoubtedly true that the Eisenstein primes at level $p_1 \cdots p_t$ deserve further study.