Baryons in Quantum Chromodynamics

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I would like to wish Dan many more productive years, during which he will continue inspiring and educating us. I am also looking forward to another chance to collaborate with Dan.

Happy Birthday !!

We consider $SU(N)$ gauge theory with N_f quarks in the fundamental representation of $SU(N)$, and N_f quarks in the anti-fundamental representation Ψ_i, $\tilde{\Psi}^i$ with $i = 1, .., N_f$. The theory has a mass parameter

$$M\Psi_i\tilde{\Psi}^i + c.c. \ ,$$

with $M \in \mathbb{C}$. We set $\theta_{QCD} = 0$ without loss of generality.

Indeed, if θ_{QCD} is present, using the ABJ "anomaly" we can show that the physics only depends on $Me^{i\theta_{QCD}/N_f}$.

The mass M is measured in units of the strong coupling scale and M is the only parameter of the theory. We will be particularly interested in the regime of $M \ll 1$, where the dynamics of the theory is richer (and there are light quarks in nature, too).

The continuous symmetries of the theory are $SU(N_f) \times U(1)_B$ (ignoring discrete factors). In particular, $U(1)_B$ acts by

$$\Psi_i \to e^{i\alpha}\Psi_i, \quad \tilde{\Psi}^i \to e^{-i\alpha}\tilde{\Psi}^i \ .$$

There are two distinct types of interesting excitations in the theory:

- particles created by local operators such as $\Psi_i \tilde{\Psi}^j$,
- particles created by local operators such as $\Psi_{i_1} \wedge \Psi_{i_2} ... \wedge \Psi_{i_N}$.

- The excitations created by $\Psi_i \tilde{\Psi}^j$ correspond to the light pseudo Nambu-Goldstone bosons (pNGBs) given by the pions. They are in the adjoint representation of $SU(N_f)$ and they are neutral under $U(1)_B$. They have zero spin. Their dynamics is described by the sigma model with target space $SU(N_f)$.
- The excitations created by $\Psi_{i_1} \wedge \Psi_{i_2} ... \wedge \Psi_{i_N}$ correspond to baryons. They carry $U(1)_B$ charge 1. They have some spin and some $SU(N_f)$ quantum numbers. Since they are completely anti-symmetric in color the baryons are therefore in

$$Sym^N \left(\frac{1}{2} \otimes \square \right) \ .$$

of $SU(2)_{spin} \times SU(N_f)$.

For instance, in nature $N = 3$ and there are three light quarks, $N_f = 3$, then $Sym^3\left(\frac{1}{2} \otimes \Box\right)$ gives baryons of spin $1/2$ in the octet of $SU(N_f = 3)$ and baryons of spin $3/2$ in the decuplet of $SU(N_f = 3)$.

An interesting twist on this story comes from 't Hooft's large N limit of QCD. 't Hooft has shown that QCD in the large N limit is a weakly coupled theory of mesons, and glueballs i.e. particles like those created by $\Psi_i\tilde{\Psi}^j$, TrF^2, etc. The weak coupling parameter is $1/N$.

$SU(N \to \infty)$ QCD $\Longleftrightarrow$ Infinitely Many Weakly Coupled Particles

The mesons are not limited to just the pNGBs, but there are many heavy mesons and the same is true for glueballs.
This begs the question where are the baryons?

Since in this limit the baryon particles are made out of a large number of quarks, one may expect that their mass scales like N. Indeed that's the case. Therefore, it is the inverse of the coupling constant $M_{Baryon} \sim 1/\hbar$, where $\hbar \sim 1/N$. We should therefore expect the baryons are solitons made out of the mesons.

This is an entirely new possible description of baryons – instead of using the fundamental quarks, we use the effective degrees of freedom (mesons and glueballs).

How can we construct solitons out of these infinitely many mesons and glueballs? Can any description of that sort be useful? The lightest particles for $M \ll 1$ are these pNGBs, living on the group manifold $SU(N_f)$

$$\mathcal{L} = N\left[Tr\left(\partial U \partial U^{-1}\right) + MTr\left(U + U^{-1}\right)\right] + \cdots \ .$$

Luckily there are configurations made out of these fields $U(x)$ which are topologically protected, since

$$\pi_3(SU(N_f)) = \mathbb{Z} \ .$$

In other words there is a conserved current

$$J_\mu = \frac{1}{24\pi^2}\epsilon_{\mu\nu\rho\sigma} Tr\left(U\partial^\nu U^{-1} U\partial^\rho U^{-1} U\partial^\sigma U^{-1}\right)$$

Actually [Veneziano; Witten], in the large N limit, the group manifold $SU(N_f)$ has to be replaced by $U(N_f)$. That does not make a big difference for what we said because the $U(1)$ factor decouples from $J_\mu = \frac{1}{24\pi^2}\epsilon_{\mu\nu\rho\sigma} Tr \left(U\partial^\nu U^{-1} U\partial^\rho U^{-1} U\partial^\sigma U^{-1}\right)$.

While small excitations of the U field are identified with the pions, the excitations for which $\int_{\mathbb{R}^3} J_0 \neq 0$ in fact carry baryon charge [Goldstone, Wilczek; Witten]. To properly understand where the quantum numbers of the baryons come from, one needs to take into account the Wess-Zumino term $\Gamma_{WZ}(U)$, related to $\pi_5(SU(N_f))$.

It is easy to see, however, that there cannot be solutions to the Euler Lagrange equations of

$$\mathcal{L} = N\left[Tr\left(\partial U \partial U^{-1}\right) + MTr\left(U + U^{-1}\right)\right]$$

which have nonzero $\pi_3(SU(N_f))$. Let us denote the typical size of the baryon by L. Then, due to the kinetic term, the energy is going to scale like L. So it is always going to be beneficial for the system to decrease the baryon size.

Since the effective action has corrections such as

$$N\left[Tr\left(\partial U\partial U^{-1}\right)+MTr\left(U+U^{-1}\right)+\frac{1}{e^2}Tr\left(\partial U\partial U^{-1}\partial U\partial U^{-1}\right)+\cdots\right]$$

One can construct solutions (Skyrmions) when including some higher-derivative terms like the one above.

There is no systematic way to decide when to truncate the expansion since the baryon stabilizes at size of order the QCD scale. Usually people just consider the four-derivative terms and then compare the properties of these baryons to data [Adkins et al...]. This generally works quite well!

One interesting special case where the the above edifice completely collapses, is $N_f = 1$.

- On the one hand, there are still baryons – indeed, $Sym^N\left(\frac{1}{2} \otimes \square\right)$ becomes just a spin $N/2$ baryon.
- On the other hand, the group manifold $U(N_f)$ becomes $U(1)$ and $\pi_3(U(1)) = 0$.

So one can ask if there is any sort of useful description of baryons (in terms of the effective degrees of freedom) in this special case. Clearly using the circle degree of freedom, $\eta' \simeq \eta' + 2\pi$, is not enough.

The effective action consists now of the η' particle alone,

$$\mathcal{L} = N\left[(\partial\eta')^2 - M\cos(\eta')\right] + \cdots$$

(We have chosen M to be real and positive.) This theory has a unique trivially gapped ground state at $\eta' = 0$.
There is an interesting conserved current with 3 indices

$$J_{\mu\nu\rho} = \frac{1}{2\pi}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma\eta' \ .$$

This is a two-form $U(1)$ symmetry (using the terminology of [Kapustin, Seiberg...]).

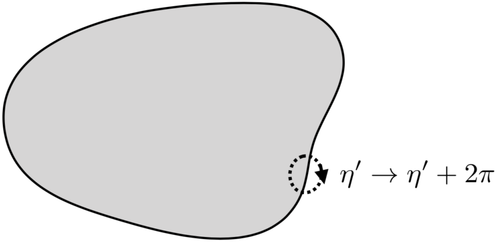
- This symmetry cannot be explicitly broken by any term in the effective theory of η'. It is also not spontaneously broken as far as the effective theory goes.
- But it is explicitly broken in the full theory, since there is no such two-form symmetry in QCD.
- The excitations which carry charge under this symmetry are sheets. Namely, 2+1 dimensional world-volumes, through which η' undergoes a monodromy on the circle. The tension of the sheet is denoted T_{sheet}. It scales linearly with N.
- It makes sense to discuss the worldvolume theory on the sheet because the probability of decay is through non-perturbative effects,

$$\Gamma \sim e^{-N} \ .$$

- Naively, the world-volume theory on the sheet is just given by the Nambu-Goto action, which is due to the spontaneously broken translational invariance

$$S_{sheet} = T_{sheet} \int d^3x \sqrt{\det \gamma_{ind}} \ .$$

- One can also imagine large sheets with a boundary. The η' field has a monodromy around that boundary.



Fortunately, the sheets are a little bit richer than what we have just described. This may seem impossible because everything we have claimed thus far follows rigorously from the effective η' theory.

The point is that the effective theory

$$\mathcal{L} = N\left[(\partial\eta')^2 - M\cos(\eta')\right] + \cdots$$

is a little misleading. There is a subleading correction [Veneziano; Witten...] which takes the form

$$N^0 min_{k\in\mathbb{Z}}(\eta' + 2\pi k)^2$$

This term in the effective action is continuous but non-differentiable at $\eta' = \pi \mod 2\pi\mathbb{Z}$.

As in any effective theory, a singularity signals that some degrees of freedom were improperly integrated out. Here, unlike in examples such as Seiberg-Witten theory, the singularity is rather mild and does not lead to new massless degrees of freedom. It is useful to think about this singularity as originating from some heavy fields jumping from one gapped vacuum to another.

If we just study small fluctuations around our vacuum $\eta' = 0$, which is what people ordinarily do, then the effect of this subtle singularity is not crucial. But our sheet excitation probes the whole range of the η' field and thus can be sensitive to some physics which is not included in the effective theory.

Cautionary Remarks

- Can this subtle phenomenon completely destroy the sheet? (namely, render it **not even** meta-stable)

In principle yes. We will assume that this does not occur.

- Is the tension still calculable?

No. Due to the jump of the heavy fields, there is an additive contribution from the potential barrier that those heavy fields had to tunnel through. The tension, however, remains of order N.

We will *propose* that the effect of this singularity is to introduce a Topological Field Theory (TFT) on the sheet. We take that to be $U(1)_N$ and denote the gauge field by a. This allows us to couple the theory to the background baryon gauge field, A^B.

$$S_{sheet} = T_{sheet} \int_{sheet} d^3x \sqrt{\det \gamma_{ind}} + \int_{sheet} \left(\frac{N}{4\pi} ada + \frac{1}{2\pi} adA^B \right) .$$

Therefore, we are finally able to couple the baryon gauge field A^B to some low-energy degrees of freedom. We could not do it just with η' because $\pi_3(U(1)) = 0$ but due to the TFT on the sheet, we are able to couple the low-energy theory to baryon number.

The conjecture about the theory on the sheet seems outlandish, but

- There is a logical connection between this story and [Gaitto, ZK, Seiberg]. The results of GKS do not imply this whole picture, but if one makes some reasonable assumptions about the phase transition described in GKS then what I say here follows.
- Recently [Aharony et al.; in progress] we have shown that all the elements above are realized in holographic constructions of QCD. In particular, there is a meta-stable sheet, it has a $U(1)_N$ TFT, and the boundary conditions are as below.
- There are rigorous results by [Armoni, Dumitrescu, Festuccia, ZK; to appear] which again strongly motivative this whole picture.

An interesting question is what to do with the action in the event that we have a boundary.

$$S_{sheet} = T_{sheet} \int_{sheet} d^3x \sqrt{\det \gamma_{ind}} + \int_{sheet} \left(\frac{N}{4\pi} ada + \frac{1}{2\pi} adA^B \right) .$$

Indeed, since the sheet is meta-stable, boundaries may appear dynamically whether we like it or not.

For the Nambu-Goto piece, the answer is quite clear. For the Chern-Simons part, there is a natural answer which fits beautifully with the microscopic theory. If we take the boundary conditions to be Dirichlet for a, then the Hilbert space has an action on it by a global $U(1)$ symmetry. That symmetry is identified with baryon number. (This is the same as in the FQHE, where baryon number is replaced by electromagnetism.) For large enough sheets, the boundary excitations are light.

So we arrived at the picture that there are excitations which carry baryon number. These excitations are constructed from edge modes of the sheet. The boundary theory is a compact, *chiral* scalar field, $\varphi \simeq \varphi + 2\pi$ [Floreanini,Jackiw...].

The configuration where φ winds around the boundary corresponds to the vertex operator

$$e^{iN\varphi} \ .$$

We can compute the spin and energy of this configuration.

The spin is computed from L_0 of that vertex operator and the energy is computed from L_0 divided by the radius of the boundary circle, L. We find

$$Spin = N/2 ,$$

$$Energy \sim N/L .$$

Amazingly, the spin works our exactly right!

The energy due to the edge mode wants to make the pancake expand. On the other hand, the tension of the sheet is trying to contract the sheet

$$E = T_{sheet} L^2 + N/L .$$

Since $T_{sheet} \sim N$, we see that the configuration wants to stabilize at $L \sim 1$, in agreement with phenomenological expectations. Plugging $L \sim 1$, we see that the mass scales linearly with N, again as expected phenomenologically.

As in the theory of Skyrmions, the fact that baryon's size is at the QCD scale means it is hard to compute things from first principles, instead one has to resort to some sort of truncation.

Let us now generalize to the case of $N_f > 1$. First, we replace $J_{\mu\nu\rho} = \frac{1}{2\pi}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma \eta'$ with

$$J_{\mu\nu\rho} = \frac{1}{2\pi}\epsilon_{\mu\nu\rho\sigma} Tr(U^{-1}\partial^\sigma U) \ ,$$

with $U \in U(N_f)$. This is just the two-form current related to $\pi_1(U(N_f))$.

We can then construct again a sheet that carries this charge and a natural conjecture is that it now the effective theory has the $U(N_f)_N$ TFT,

$$\int_{sheet} \left(\frac{N}{4\pi} Tr\left(ada + \frac{2}{3}a^3 \right) + \frac{1}{2\pi} Tr(a) dA^B \right)$$

The most natural boundary conditions are again Dirichlet. They lead to a Hilbert space which furnishes a representation of $U(N_f)$ which is exactly the symmetry group of QCD!
We can ask about the edge modes again, and in particular about the simplest edge mode which carries baryon charge. The properties of this excitation can be computed using the formalism of [Moore,Seiberg]

- It has spin $N/2$,
- It is in the $Sym^N(\square)$ representation of $SU(N_f)$,
- It has (as before) size of order 1 and mass of order N.

These precisely agrees with the properties of the maximal spin baryon created by $\Psi_{i_1} \wedge \Psi_{i_2} \ldots \wedge \Psi_{i_N}$.

Qualitatively, these baryons look flattened and the baryon density peaks in a solid torus, around the edge of the disc. Does it have anything to do with how baryons really look like?

For one, these baryons have a very large spin, of order N, so there are strong spin-orbit and spin-spin forces (this is unlike low-spin baryons, which the Skyrmion model rightly predicts are approximately spherical). One may expect these baryons to be spatially deformed.

In addition, since all the spins are aligned, the spin-spin repulsion leads to the quarks trying to get far away from each other. So it is not inconceivable that they would end up on the boundary of a disc.

This leads one to suspect that this model, for $N_f > 1$, is not in contradiction to the Skyrmion model, rather, it is complementary. This picture of the baryons as edge excitations of the droplet is probably best suited for high-spin baryons.

One can ask, where are the other baryons? say, the one with $Spin = N/2 - 1$. There is no such excitation of the Hilbert space on a disc. But following the above intuition, since one spin is not aligned any longer, we can squeeze one quark to be in the middle of the pancake, as it has no energetic reasons to be on the edge. Repeating the analysis with one quark in the bulk of the disc, we find precisely the sought-after excitation! [Choi et al.; in progress]

- We have proposed that QCD in the large N limit admits exponentially long-lived sheets which carry the TFT $U(N_f)_N$.
- The sheets may have a boundary and there are excitations then which carry baryon charge. They are made entirely of the effective fields on the sheet. This leads to a model of baryons.
- These baryons are naturally fast spinning. Their properties beautifully agree with what we expect phenomenologically.
- We suggest that this model is complementary to the Skyrmion model. Our model makes sense also for $N_f = 1$, when the Skyrmion model does not.

Many open questions...

- Are these agreements in quantum numbers and other qualitative properties a coincidence?
- A detailed description of baryons of spin $N/2 - r$ is yet to be worked out. Many new consistency checks.
- Holographic model?
- Contact with phenomenology at $N = 3$?
- Many other gauge theories raise similar questions and so we expect nontrivial generalizations.
- Expect these sheets will be important in nuclear matter, e.g. when we have finite chemical potential for baryon charge and angular momentum.
- $\dots$