Higgs mechanism

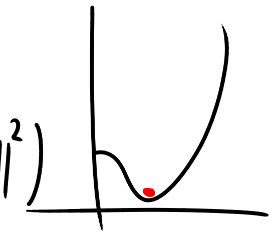
Consider a gauge theory coupled to matter: $\mathcal{C} = \begin{cases} (P,\nabla): & \text{principal } G\text{-bdle w/ conn} \\ \phi: & \text{section of } V_P \end{cases}$

$$S = \frac{1}{4g^2}\int_X F\wedge * F + \frac{1}{2}\int_X \|D\phi\|^2 + \int_X H(\phi)$$

where H is some G-invariant function.

Suppose moreover that H has minima other than the origin.
For simplicity, say H has a single G-orbit worth of minima.

e.g. $G = SU(2)$, $V =$ fundamental, $H(\phi) = \left(\|\phi\|^4 - 2m^2\|\phi\|^2\right)$



In this case, the perturbative physics at low energies will be best understood as an expansion around the orbit $\|\phi\| = m$, not around $\phi = 0$.

In this pert. expansion, we can gauge-fix by taking P trivial and choosing say $\phi = \begin{pmatrix}1\\0\end{pmatrix} f$, f real. Then our theory is $\approx$ to one with $\mathcal{C} = \begin{cases} \text{conn } A \text{ on triv. } P \\ \phi = \begin{pmatrix}1\\0\end{pmatrix} f \end{cases}$ and no gauge group $\mathcal{G}$ anymore.

Write $\phi_0 = m\begin{pmatrix}1\\0\end{pmatrix}$ and $\phi = \phi_0 + \delta\phi$;

then $\|D\phi\|^2 = \|D(\phi_0+\delta\phi)\|^2 = \|A\phi_0\|^2 + (\text{terms involving } \delta\phi)$

Let's see concretely what $\|A\phi_0\|^2$ looks like:

if $A = A^1\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + A^2\begin{pmatrix}0 & i\\ i & 0\end{pmatrix} + A^3\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$, $\quad A^i \in \Omega^1(X)$

get $\|A\phi_0\|^2 = m^2\left(\|A^1\|^2 + \|A^2\|^2 + \|A^3\|^2\right)$

i.e. all 3 components of A become massive. At energies $E \ll gm$ they should be integrated out. $\delta\phi$ is also massive.

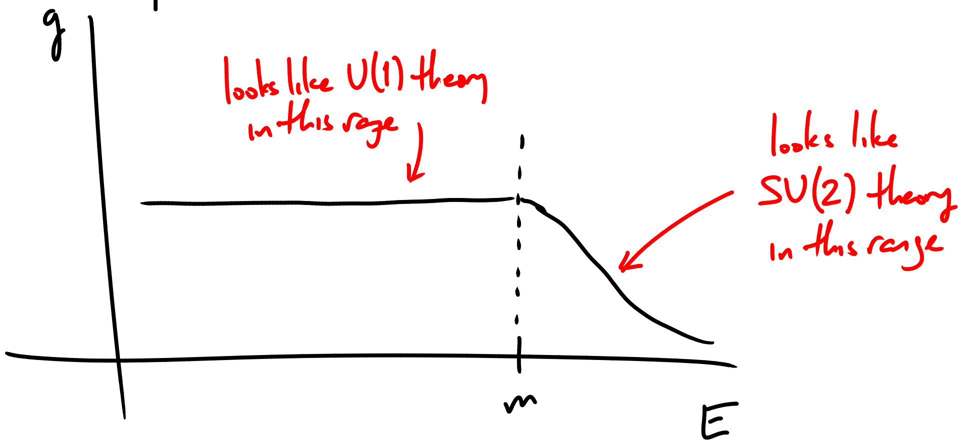
If instead we take $V =$ adjoint, then the picture is different: fix $\phi = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} f$ say, then we have a residual gauge symmetry left over:

$$\mathcal{G}_{eff} = \mathrm{Map}\{X, Z(\phi_0)\} \simeq \mathrm{Map}\{X, U(1)\}$$

Have $\|[A,\phi_0]\|^2 = m^2 \left(\|A^2\|^2 + \|A^3\|^2 \right)$ so only 2 of the 3 components of A become massive. ("W bosons")

Set massive fields to $0 \implies S_{eff} = \frac{1}{4g^2} \int F^1 \wedge * F^1$

How reliable is this picture?
It would predict that the couplings run like:



It will be most reliable when $m \gg \Lambda_s$ with Λ_s the strong-coupling scale of the nonabelian theory. (In that case the effective coupling is weak everywhere along the flow.)

For $m \lesssim \Lambda_s$, quantum corrections can change the picture even qualitatively.