Anomalies

The action we wrote had a symmetry $U(1)_R$.

Observables $\mathcal{O}^{(k)}$ have $U(1)_R$ charge $4-k$.

So naively we would expect that $\left\langle \prod_i \mathcal{O}^{(k_i)}(\gamma_i) \right\rangle = \left\langle \prod_i e^{i\vartheta(4-k_i)} \mathcal{O}^{(k_i)}(\gamma_i) \right\rangle$

and hence $\left\langle \prod_i \mathcal{O}^{(k_i)}(\gamma) \right\rangle = 0$!

Fortunately this is wrong — for an interesting reason: the $U(1)_R$ symmetry formally present in the action is not actually present in the path integral.

To see this, consider the cutoff theory, in the sector with $\frac{1}{8\pi^2}\int \mathrm{Tr}\, F\wedge F = k$.

The subtlety comes from the fermion sector. $U(1)_R$:
$$\chi \to e^{-i\vartheta}\chi \qquad \psi \to e^{i\vartheta}\psi \qquad \eta \to e^{-i\vartheta}\eta$$

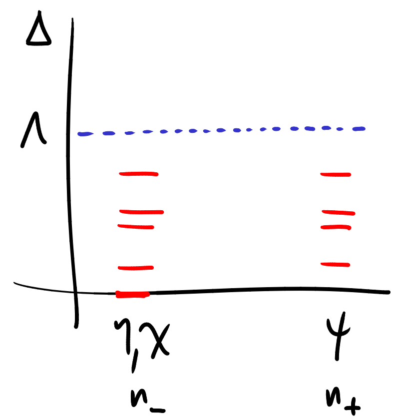
Naively, $\mathcal{D}\chi\,\mathcal{D}\psi\,\mathcal{D}\eta$ looks like it would be invariant (ψ has 4 cpts, χ has 3, η has 1)

But, look closer: quadratic terms for fermions $\langle \psi, L(\eta+\chi)\rangle$ $\quad L = d_\nabla + (d_\nabla^+)^*$

$$\underset{\eta}{\Omega^0(\mathfrak{g}_{\mathbb{C},P})} \oplus \underset{\chi}{\Omega^2(\mathfrak{g}_{\mathbb{C},P})} \underset{L^*}{\overset{L}{\rightleftarrows}} \underset{\psi}{\Omega^1(\mathfrak{g}_{\mathbb{C},P})} \qquad\qquad \{L, L^*\} = \Delta \qquad (*)$$

In the cutoff theory we integrate over fields with Δ-eigenvalue less than Λ.

We've studied reps. of $(*)$ before: fields with $\Delta \neq 0$ come in pairs, but those with $\Delta = 0$ need not be paired.



$$I = n_+ - n_- = \mathrm{index}(L) = -2p_1(V) - \tfrac{3}{2}(\chi + \sigma)$$

So: $\left\langle \prod_i \mathcal{O}^{(k_i)}(\gamma_i) \right\rangle = e^{-i\mathcal{I}\vartheta} \left\langle \prod_i e^{i\vartheta(4-k_i)} \mathcal{O}^{(k_i)}(\gamma_i) \right\rangle$

$\quad\quad\quad \parallel$

$\quad\quad\quad 0$ <u>unless</u> $\mathcal{I} = \sum_i 4-k_i$