

## Laplacians

Def  $X$  almost  $\mathbb{C}$  with Hermitian metric:  $\partial^* = -\star \circ \bar{\partial} \circ \star : \Omega^{p,q}(X) \rightarrow \Omega^{p-1,q}(X)$   
 $\bar{\partial}^* = -\star \circ \partial \circ \star : \Omega^{p,q}(X) \rightarrow \Omega^{p,q-1}(X)$

Lemma  $X$  cplx  $\Rightarrow d^* = \partial^* + \bar{\partial}^*$ ,  $(\partial^*)^2 = (\bar{\partial}^*)^2 = 0$

Pf Just use  $d = \partial + \bar{\partial}$ ,  $\partial^2 = \bar{\partial}^2 = 0$ .  $\blacksquare$

Def  $X$  cplx with Hermitian metric:  $\Delta_{\partial} = \partial^* \partial + \partial \partial^*$ ,  $\Delta_{\bar{\partial}} = \bar{\partial}^* \bar{\partial} + \bar{\partial} \bar{\partial}^*$ .