

# MODULI OF HIGGS BUNDLES

## 1. SYLLABUS

Moduli spaces of Higgs bundles are extremely geometrically rich. The aim of the course is to learn as much as we can about the topology and geometry of these spaces. One idiosyncratic point is that we will focus more than usual on the fact that these spaces are *hyperkähler* (as opposed to merely holomorphic symplectic), and our point of view will be more differential-geometric than algebraic-geometric.

A list of possible topics (too much to really cover, so some of the things with question marks will have to be dropped; also some things will probably have to be done in a different order):

- speedy review of basic definitions in complex and Kähler geometry ✓
- basic notions of hyperkähler geometry ✓
- twistor spaces (Hitchin's theorem) ✓
- hyperholomorphic bundles
- examples of hyperkähler spaces:
  - $\mathbb{R}^4, \mathbb{R}^3 \times S^1, \mathbb{R}^2 \times T^2$  ✓
  - Gibbons-Hawking spaces ✓
  - ALE spaces ✓
  - Ooguri-Vafa manifold ✓
  - Atiyah-Hitchin manifold?
  - complex coadjoint orbits?
  - moduli space of instantons?
  - moduli space of solutions of Nahm equations?
  - cotangent bundles of Riemann surfaces (incomplete)?
  - hypertoric varieties?
  - K3 surface ✓
- symplectic and hyperkähler quotients ✓
- moduli space of Higgs bundles for the group  $G = U(1)$  (incl. speedy review of abelian Hodge theory) ✓
- moduli spaces of semistable  $G$ -Higgs bundles ✓
- Narasimhan-Seshadri theorem ✓
- Hitchin's integrable system, spectral curves ✓
- SYZ mirror symmetry?
- nonabelian Hodge theorem ✓
- hyperkähler structure on moduli of Higgs bundles ✓
- Morse function, computation of Betti numbers
- asymptotic construction of harmonic bundles and hyperkähler metric
- bundles with parabolic structure
- abelianization, cluster coordinates?
- exact (conjectural) description of hyperkähler metric?
- $P = W$  conjecture?

- role in geometric Langlands program?
- role in  $\mathcal{N} = 2$  supersymmetric QFT?
- holomorphic disc counting and GW/DT-type invariants?
- $S$ -duality action on branes?
- Hitchin's section (higher Teichmuller space)?