

Linearity principle

E, F vector bundles over M .

Fact If $\varphi: \mathcal{E}(E) \rightarrow \mathcal{E}(F)$ is linear over $C^\infty(M)$

(i.e. $\varphi(fs) = f\varphi(s) \quad \forall s \in \mathcal{E}(E), f \in C^\infty(M)$)

then $\exists \gamma \in \mathcal{E}(\text{Hom}(E, F))$ s.t. $\varphi(s) = \gamma \cdot s$.

Pf Fix a basis for E , then determine γ by $\gamma \cdot e_a = \varphi(e_a)$

C^∞ -linearity then $\Rightarrow \varphi = \gamma$