Lecture 19

15 Oct 2012

Last time: Double integrals over general regions

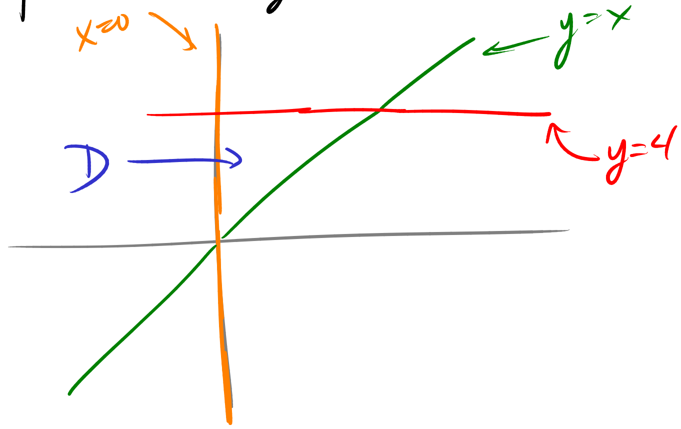
We choose the order of $\int$ in order to make the integral simplest.

Ex Let D be the region in the xy-plane bounded by the 3 lines:

$y = x$

$y = 4$

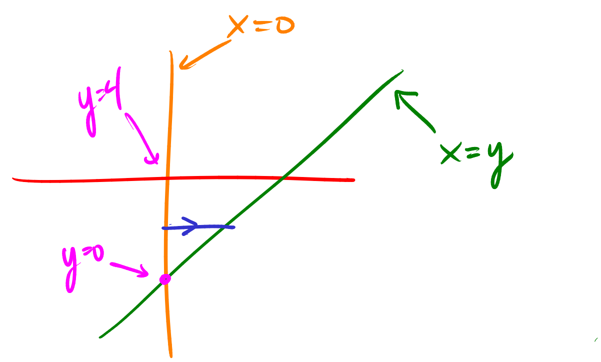
$x = 0$



Calculate $\displaystyle\iint_D y^2 e^{xy}\, dA.$

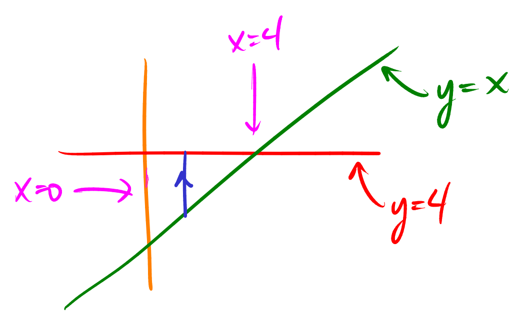
If we slice horizontally:

$$\int_0^4 \left[\int_0^y y^2 e^{xy}\, dx\right] dy$$



If we slice vertically:

$$\int_0^4 \left[\int_x^4 y^2 e^{xy}\, dy\right] dx$$



The horiz. slicing looks easier — let's use that:

$$\int_0^4 \left[\int_0^y y^2 e^{xy}\, dx\right] dy$$

$$= \int_0^4 \left(y^2 \cdot \frac{1}{y} e^{xy} \Big]_{x=0}^{x=y} \right) dy$$

$$= \int_0^4 \left(y e^{xy} \Big|_{x=0}^{x=y} \right) dy$$

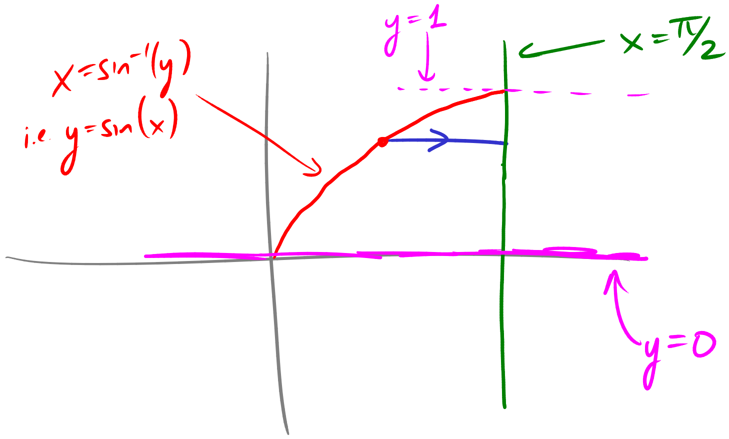
$$= \int_0^4 y e^{y^2} - y \, dy$$

$$= \frac{1}{2} e^{y^2} - \frac{1}{2} y^2 \Big|_0^4 = \left(\frac{1}{2} e^{16} - 8 \right) - \left(\frac{1}{2} \right) = \frac{1}{2} e^{16} - \frac{17}{2}$$

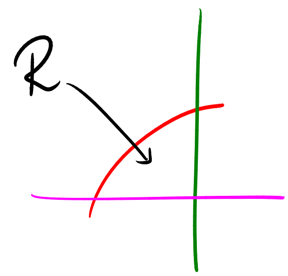
Ex Compute the iterated integral

$$\int_0^1 \int_{\sin^{-1}(y)}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\ dx\, dy$$

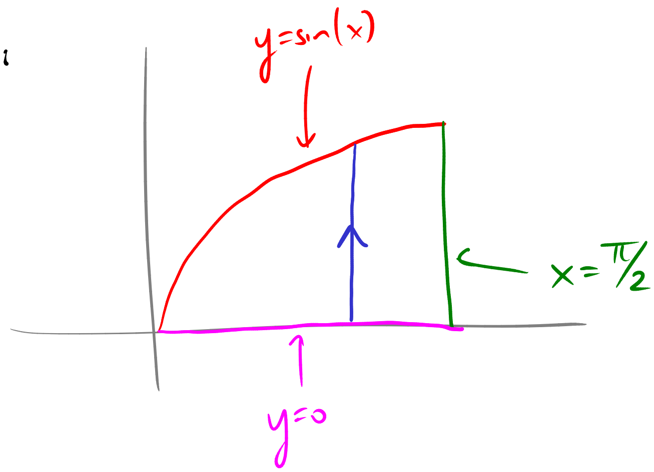
Looks hard — let's see if it becomes easier when we reverse the order of integration.



Our iterated $\int$ is the same as $\displaystyle\iint_R \cos x\sqrt{1+\cos^2 x}\ dA$



Try slicing it vertically:



$$\int_0^{\pi/2} \left[\int_0^{\sin(x)} \cos x \sqrt{1+\cos^2 x}\ dy \right] dx$$

$$= \int_0^{\pi/2} \left(\left(\cos x\sqrt{1+\cos^2 x}\right) y \Big]_0^{\sin(x)} \right) dx$$

$$= \int_0^{\pi/2} \sin x \cos x \sqrt{1+\cos^2 x}\ dx$$

$u = \cos x$

$du = -\sin x\, dx$

$$= -\int_1^0 u\sqrt{1+u^2}\,du \qquad z = 1+u^2$$

$$= \dots$$

$$= \frac{1}{3}\left(2^{3/2} - 1\right)$$

<u>Why not</u> just

$$\int_0^1 \int_{\sin^{-1}(y)}^{\pi/2} f(x,y)\,dx\,dy$$

$$= \int_{\sin^{-1}(y)}^{\pi/2} \int_0^1 f(x,y)\,dy\,dx \qquad \text{wrong}$$

You can see this is wrong b/c the answer shouldn't depend on a variable... should be just a number!

For double $\int$ over a rectangle we can easily reverse the limits, i.e.

$$\int_b^a \int_d^c f(x,y)\,dx\,dy = \int_d^c \int_b^a f(x,y)\,dy\,dx$$

But for double $\int$ over a more complicated shape, we really need to draw the picture to see how to reverse the order.
