# Lecture 40

7 Dec 2012

Review session  Tue 12–3pm  Room to be determined

Final exam  Fri 7–10pm

- 24 problems  ~ 8 from each third of the course

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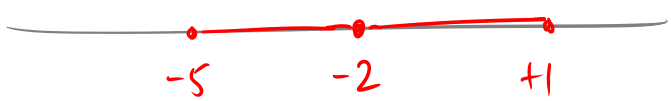
Interval of convergence: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x+2)^n}{3^n (n+1)}$

Ratio test: $\displaystyle\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x+2|^{n+1}}{3^{n+1}(n+2)} \cdot \frac{3^n (n+1)}{|x+2|^n} = \frac{|x+2|}{3} \cdot \frac{n+1}{n+2} \longrightarrow \frac{|x+2|}{3}$

$$\frac{|x+2|}{3} < 1 \iff |x+2| < 3$$




At $-5$: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(-3)^n}{3^n(n+1)} = \sum_{n=0}^{\infty} \frac{1}{n+1}$  div. (p-test, lim-comp)

At $1$: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(3)^n}{3^n(n+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$  conv. (alt. series)

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$$\lim_{n\to\infty} n^{1/n} = 1.$$

Why? Take log of both sides:

$$\lim_{n\to\infty} n^{1/n} = L$$

$$\lim_{n\to\infty} \log(n^{1/n}) = \log(L)$$

$$\lim_{n\to\infty} \frac{\log(n)}{n} = \log(L)$$

$$\lim_{n\to\infty} \frac{\frac{1}{n}}{1} = \log(L)$$

$$0 = \log(L)$$

$$1 = L$$

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$$\int \frac{1}{\sqrt{9x^2+36x+117}}\,dx$$

Want to rewrite the bottom eliminating the linear part $(\cdot)x$

$$= \int \frac{dx}{3\sqrt{x^2+4x+13}}$$

$$= \int \frac{dx}{3\sqrt{(x+2)^2+9}}$$

$u = x+2$

$du = dx$

$$= \int \frac{du}{3\sqrt{u^2+9}}$$

$u = 3\tan\theta$

$du = 3\sec^2\theta\, d\theta$

$$= \int \frac{3\sec^2\theta\, d\theta}{3\sqrt{9\tan^2\theta+9}}$$

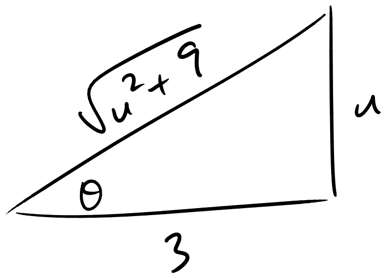
$\tan^2\theta + 1 = \sec^2\theta$

$$= \frac{1}{3}\int \frac{\sec^2\theta\, d\theta}{\sec\theta}$$

$$= \frac{1}{3}\int \sec\theta\, d\theta = \frac{1}{3}\ln|\sec\theta+\tan\theta| + C$$

$\tan\theta = \frac{u}{3} = \frac{x+2}{3}$

$u = x+2$
$du = dx$




$u = 3\tan\theta$
$du = 3\sec^2\theta \, d\theta$

$\sec\theta = \frac{hyp}{adj} = \frac{\sqrt{u^2+9}}{3}$

So $\int = \frac{1}{3}\ln\left|\frac{\sqrt{(x+2)^2+9}}{3} + \frac{x+2}{3}\right| + C$

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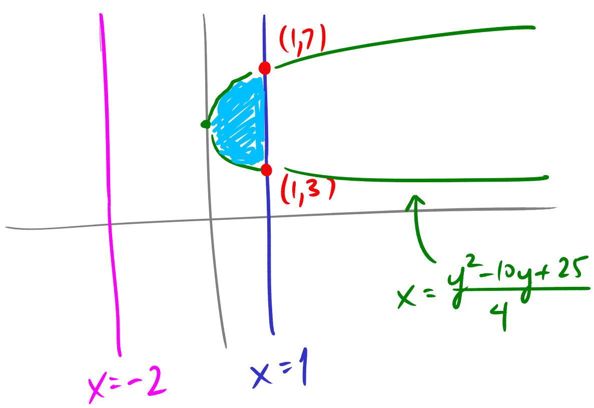
## Volumes:

Find volume of solid obtained by rotating region between $x=1$ and $4x = y^2-10y+25$ around the line $x=-2$.




Find intersections:

$4 = y^2-10y+25$

$0 = y^2-10y+21$

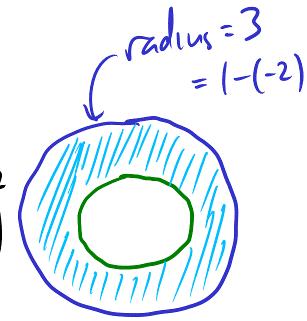
$= (y-3)(y-7)$

So int's are at $(1,3)$ $(1,7)$

Plug in $y=5$: $4x = 25-50+25 = 0$ i.e. $x=0$

Slice by slices of fixed $y$: $V = \int_3^7 A(y) \, dy$

Cross-sections look like washers: $A(y) = \pi(3^2) - \pi\left(\frac{y^2-10y+25}{4}+2\right)^2$



So $V = \int_3^7 9\pi - \left(\frac{y^2-10y+25}{4} + 2\right)^2 \pi \quad = \cdots$

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$\sin(A+B) = \cdots$

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Find Taylor series for $\ln(x)$ centered at $x=3$.

Use formula for Taylor series — $\displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

$$f(x) = \ln x$$

$$f'(x) = \frac{1}{x}$$

$$f''(x) = -\frac{1}{x^2}$$

$$f'''(x) = \frac{2}{x^3}$$

$$f^{(4)}(x) = -\frac{6}{x^4}$$

$$\vdots$$

$$f^{(n)}(x) = (-1)^{n+1}\frac{(n-1)!}{x^n} \qquad (n \geq 1)$$

So, Taylor series is $\displaystyle \ln 3 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-3)^n}{n3^n}$

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