Q: Find the Taylor series for $f(x) = \frac{1}{x}$ centered at $x = -3$.

A1:

$$f(x) = \frac{1}{x}$$

$$f'(x) = -\frac{1}{x^2}$$

$$f''(x) = \frac{2}{x^3}$$

$$f'''(x) = -\frac{6}{x^4}$$

$$\vdots$$

$$f^{(n)}(x) = (-1)^n \frac{n!}{x^{n+1}} \qquad f^{(n)}(-3) = (-1)^n \frac{n!}{(-3)^{n+1}} = -\frac{n!}{3^{n+1}}$$

So Taylor series is $\displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(-3)}{n!} \cdot (x+3)^n$

$$= \sum_{n=0}^{\infty} -\frac{n!}{3^{n+1} n!} (x+3)^n$$

$$= \underline{\underline{\sum_{n=0}^{\infty} -\frac{(x+3)^n}{3^{n+1}}}}$$

A2: $\displaystyle\frac{1}{x} = \frac{1}{(x+3)-3} = -\frac{1}{3} \frac{1}{1-\frac{x+3}{3}} = -\frac{1}{3} \sum_{n=0}^{\infty} \left(\frac{x+3}{3}\right)^n$

$$= \underline{\underline{-\sum_{n=0}^{\infty} \frac{(x+3)^n}{3^{n+1}}}}$$