

Q: How to simplify factorials?

Like $\frac{(2n+2)!}{(2n)!}$ say.

A: First, $k!$ means a product, $k! = k \cdot (k-1) \cdot (k-2) \cdot \dots \cdot (1)$
[e.g. $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$]

$$\text{So, } (2n+2)! = (2n+2) \cdot (2n+2-1) \cdot (2n+2-2) \cdot \dots \cdot (1) \\ = (2n+2)(2n+1)(2n) \dots (1)$$

$$\text{while } (2n)! = (2n)(2n-1)(2n-2) \dots (1)$$

Dividing the two,

$$\frac{(2n+2)!}{(2n)!} = \frac{(2n+2)(2n+1)(2n)(2n-1) \dots (1)}{(2n)(2n-1)(2n-2)(2n-3) \dots (1)}$$

$$= \frac{(2n+2)(2n+1)(2n)(2n-1) \dots (1)}{(2n)(2n-1) \dots (1)}$$

$$= \frac{(2n+2)(2n+1) \cancel{(2n)} \cancel{(2n-1)} \dots \cancel{(1)}}{\cancel{(2n)} \cancel{(2n-1)} \dots \cancel{(1)}}$$

$$= (2n+2)(2n+1) \leftarrow \text{a much simpler formula}$$