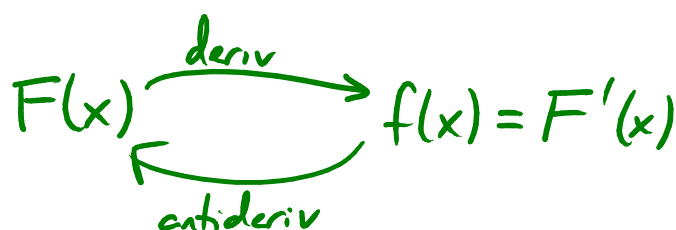## Housekeeping:

HW02 due 3am 1/26 (this Tue)
HW01 due 3am 1/29 (this Fri)

- Some of the problems are harder than the examples I show in lecture!
- You can (and should) enter your answers into QUEST as you go, rather than all at once when you finish.

Last time

Antiderivative

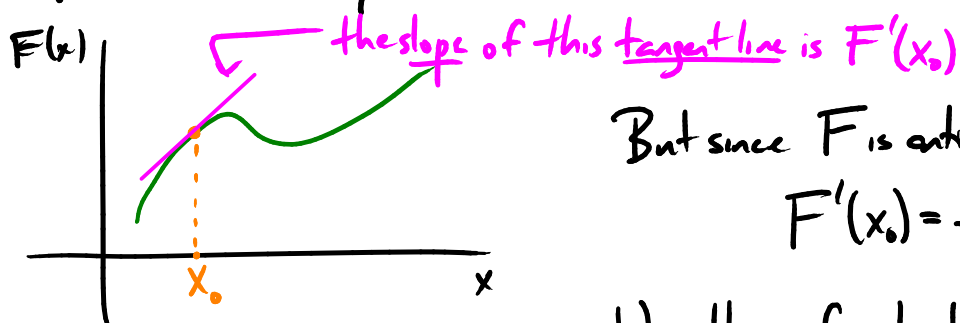$F(x) \xrightarrow{\text{deriv}} f(x) = F'(x)$ antideriv

Erratum: The most general antideriv. of $f(x) = x^n$ is $\frac{1}{n+1} x^{n+1} + C$ except when $n = -1$!

The most general antideriv. of $f(x) = x^{-1} = \frac{1}{x}$ is $F(x) = \ln x + C$

More about antideriv (Ch 4.9)

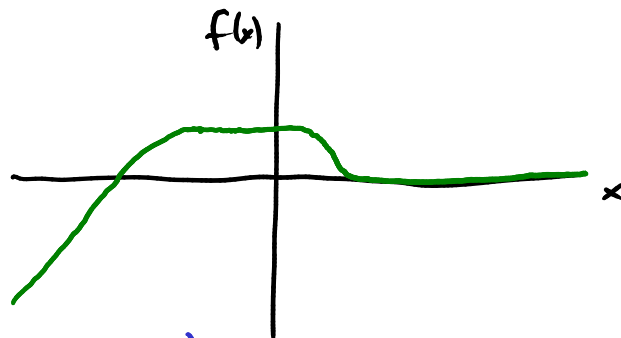Suppose we don't know a formula for $f(x)$, but we do have its graph. What can we say about its antideriv. $F(x)$?

the slope of this tangent line is $F'(x_0)$
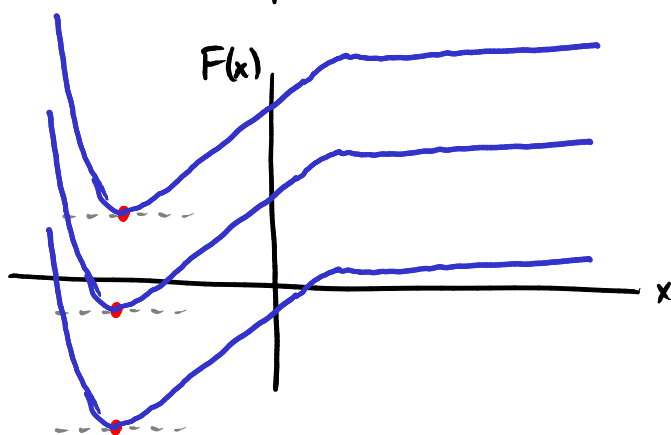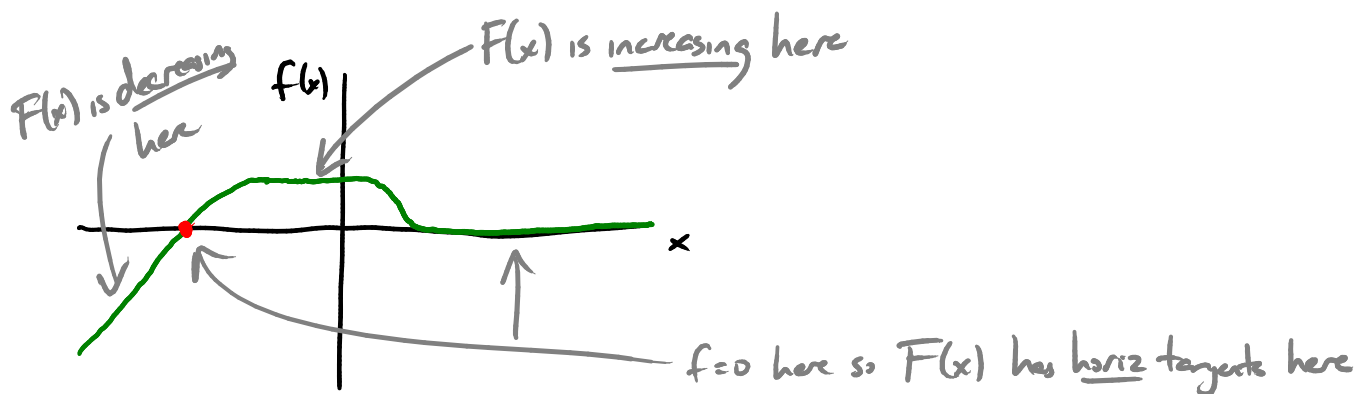
But since F is antideriv. of f,

$$F'(x_0) = f(x_0)$$

Use this info. about the tangent lines to sketch F.

Ex. Suppose the graph of $f(x)$ looks like

$f(x)$

Sketch the graphs of a few possible antideriv. $F(x)$.

$F(x)$ is decreasing here

$F(x)$ is increasing here

$f(x)$

$f = 0$ here so $F(x)$ has horiz tangents here
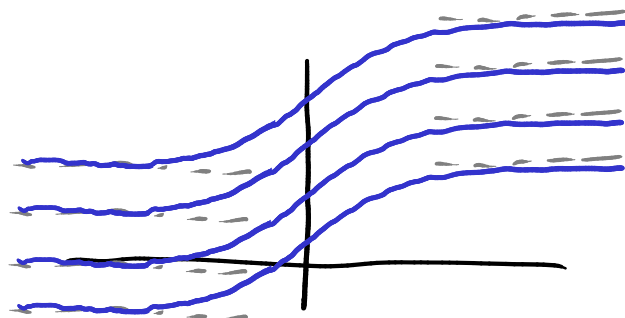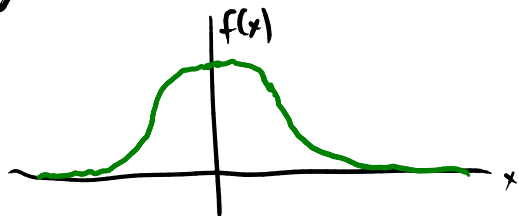
$F(x)$

An aside:

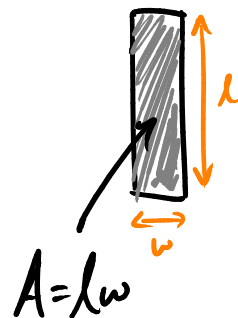Every continuous function $f(x)$ [and even some discontinuous ones...] has an antiderivative $F(x)$.

But sometimes $F(x)$ can't be written in terms of "elementary" functions — even though $f(x)$ can!
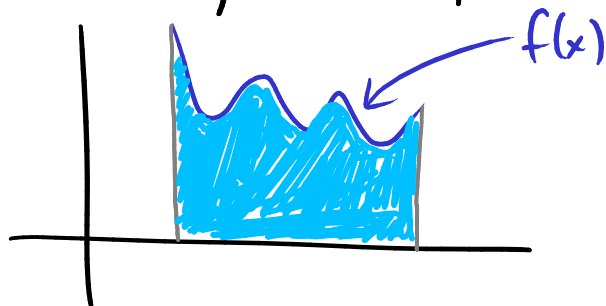
e.g. $f(x) = e^{-x^2}$.

$f(x)$

Areas (Ch 5.1)

We all know the areas of simple shapes

$$A = l\omega$$

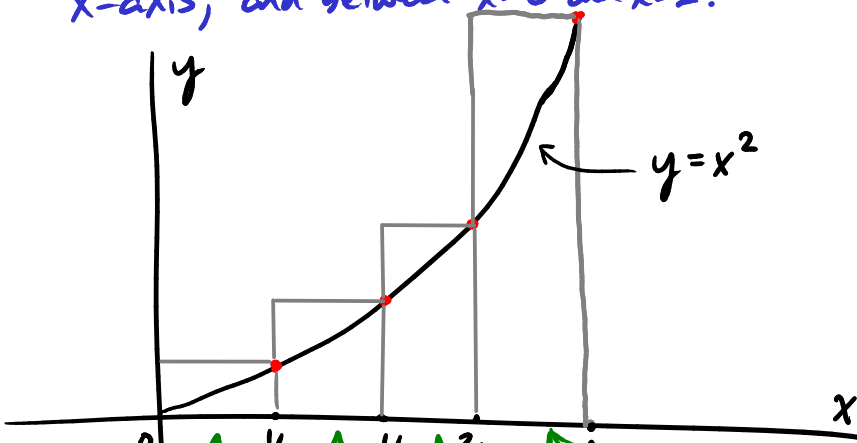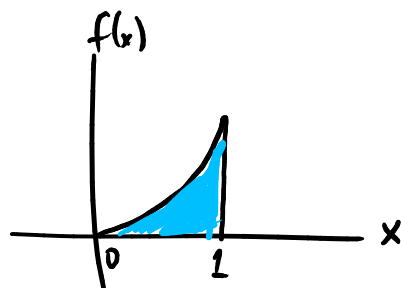How about a more complicated shape?

$f(x)$

Could try to estimate the area by "counting boxes" (put the figure on graph paper).
Not exact, but if the boxes are very small, it's close.
The exact answer will be the limit of our estimate as the size of the boxes goes to zero.

Ex. Say $f(x) = x^2$

$f(x)$

Estimate the area of the region between $y = f(x)$ and the x-axis, and between $x = 0$ and $x = 1$.
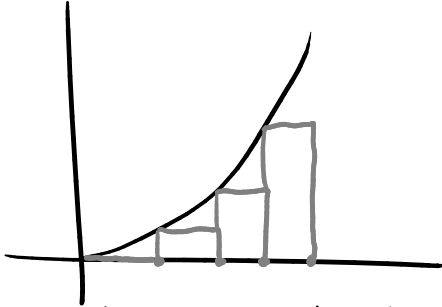
$y = x^2$

$$A = \frac{1}{4} \cdot \left(\frac{1}{4}\right)^2 \qquad A = \frac{1}{4} \cdot \left(\frac{1}{2}\right)^2 \qquad A = \frac{1}{4} \cdot \left(\frac{3}{4}\right)^2 \qquad A = \frac{1}{4} \cdot (1)^2$$

total estimated area
$$A = \frac{1}{4}\left[\left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)^2 + 1^2\right]$$

simplify ⟶ $A = \dfrac{15}{32}$

This is the "estimated area using 4 rectangles of equal widths and using the right endpoints as the sample pts."

If we used the **left** endpts

we will get a different estimated area (next time).