Lecture 6

1 Feb 2010

Last time: Fund^l Theorem of Calculus I, II

I: $\int_a^x f(x)\,dx$ is an antiderivative of $f(x)$.

i.e. $\dfrac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

II: $\int_a^b f(x)\,dx = F(b) - F(a) = F\Big|_a^b$

where F is any antideriv. of f.

Indefinite integrals

Notation: $\int f(x)\,dx$ means any antiderivative of $f(x)$.

Ex: $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$

$\int \sin x\,dx = -\cos x + C$ $\quad$ $\int \cos x\,dx = \sin x + C$

$\int \sec^2 x\,dx = \tan x + C$ $\quad$ $\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$ $\quad$ $\int \csc x \cot x = -\csc x + C$

Ex Find $\int (10x^4 + 6\sec^2 x)\,dx$

$= 10\left(\dfrac{x^5}{5}\right) + 6\tan x + C = \underline{\underline{2x^5 + 6\tan x + C}}$

Ex Find $\int_0^{\pi/4} (10x^4 + 6\sec^2 x)\,dx$

$= 2x^5 + 6\tan x \Big|_0^{\pi/4}$

$$= \left[2\left(\frac{\pi}{4}\right)^5 + 6\tan\left(\frac{\pi}{4}\right)\right] - \left[2(0)^5 + 6\tan(0)\right]$$

$$= \frac{\pi^5}{512} + 6 - 0$$

$$= \frac{\pi^5}{512} + 6$$

Ex Find $\int u^{2/3}\,du$. $n = \frac{2}{3}$ $n+1 = \frac{5}{3}$

$$= \frac{3}{5}u^{5/3} + C$$

Ex Find $\int_1^8 u^{2/3}\,du$.

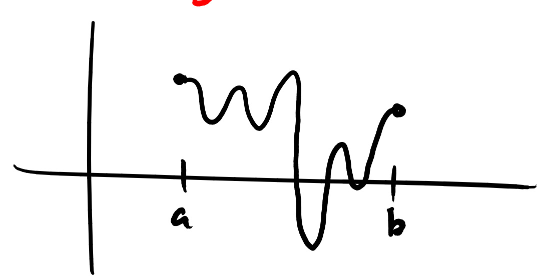
$$= \frac{3}{5}u^{5/3}\Big|_1^8 = \frac{3}{5}\left(8^{5/3} - 1^{5/3}\right)$$

$$= \frac{3}{5}(32-1) = \frac{93}{5}$$

Net change (Ch 5.4)

F' = the rate of change of F.

$\int_a^b F'(x)\,dx = F(b) - F(a)$ = net change of F over $[a,b]$.



e.g. Water flows into a reservoir at the rate $(10t+6)$ ft^3/s (t in seconds)

The reservoir contains 400 ft^3 of water at $t=0$.

How much does it have at time $t = 10$ s?

The <u>net change</u> from $t=0$ to $t=10$ is

$$\int_0^{10} (10t + 6)\, dt = 5t^2 + 6t \Big|_0^{10}$$

$$= \left[5(10^2) + 6(10)\right] - \left[5(0^2) + 6(0)\right]$$

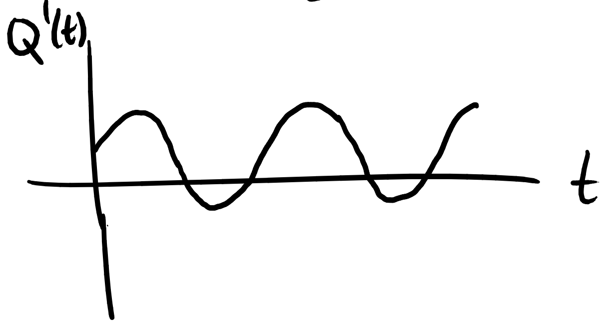
$$= 560 \text{ ft}^3$$

So the amount of water at $t = 10$s is $400 + 560 = \underline{\underline{960 \text{ ft}^3}}$

e.g. A rechargeable battery is connected to a load that can <u>charge</u> or <u>discharge</u> it. The current flowing <u>into</u> the battery is

$$Q'(t) = \sin(\pi t) + \frac{1}{2}$$

$$\left[Q(t) = \text{the charge of the battery}\right]$$



If the battery starts with 10 units of charge at $t=0$ $\left[Q(0) = 10\right]$ how much does it have at $t = 6$?

$$Q(t=6) - Q(t=0) = \int_0^6 Q'(t)\, dt$$

$$= \int_0^6 \left(\sin(\pi t) + \frac{1}{2}\right) dt$$

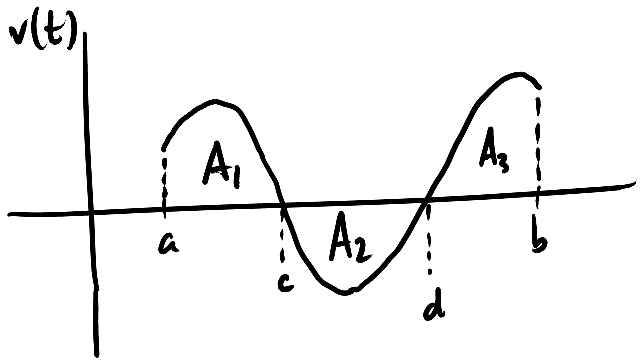
$$= -\frac{1}{\pi}\cos(\pi t) + \frac{t}{2} \Big|_0^6$$

$$= \left(-\frac{1}{\pi}\times 1 + \frac{6}{2}\right) - \left(-\frac{1}{\pi}\times 1 + \frac{0}{2}\right)$$
$$= -\frac{1}{\pi} + 3 + \frac{1}{\pi}$$
$$= 3$$

$$Q(t=6) = 3 + Q(t=0) = 3 + 10 = \underline{\underline{13}}$$

A standard example of net change: <u>total displacement</u>.

Remember if $s(t)$ = position [along some line]
$s'(t) = v(t)$ velocity



[$v(t) > 0$: $s(t)$ <u>increasing</u> i.e. moving to the <u>right</u>

$v(t) < 0$: $s(t)$ <u>decreasing</u> i.e. moving to the <u>left</u>]

<u>Total displacement</u> $\quad s(b) - s(a) = \int_a^b v(t)\,dt = A_1 + A_3 - A_2$

<u>Total distance</u> $\quad A_1 + A_2 + A_3 = \int_a^b |v(t)|\,dt$

$$= \int_a^c v(t)\,dt + \int_c^d -v(t)\,dt + \int_d^b v(t)\,dt$$

<u>Ex</u> A particle moves along a line with $v(t) = t^2 - t - 6$ m/s. (t in sec) from time $t=1$ to $t=4$.

a) What is the total <u>displacement</u> of the particle?

$$\Delta s = s(4) - s(1) = \int_1^4 v(t)\,dt = \int_1^4 t^2 - t - 6\,dt$$

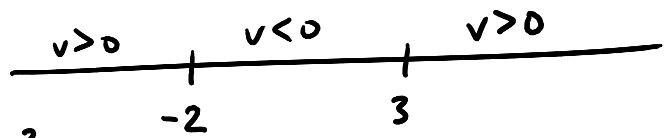
$$= \frac{t^3}{3} - \frac{t^2}{2} - 6t \Big|_1^4$$

$$= -\frac{9}{2} \text{ m} \qquad \left(\text{i.e. } \frac{9}{2} \text{ m to the } \underline{\text{left}}\right)$$

b) What is the total <u>distance</u> the particle covers?

$$\int_1^4 |v(t)| \, dt$$

$$v(t) = (t-3)(t+2)$$



$$\int_1^4 |v(t)| \, dt = \int_3^4 v(t) \, dt + \int_1^3 -v(t) \, dt$$

$$= \cdots$$