Ex $\int_0^3 \frac{5x^2+10x+2}{10x^2+4}\,dx$ $\quad U = 10x^2+4$

$du = 20x\,dx \quad dx = \frac{du}{20x}$

$= \int_0^3 \frac{5x^2+2}{10x^2+4}\,dx + \int_0^3 \frac{10x}{10x^2+4}\,dx$

$= \int_0^3 \frac{5x^2+2}{2(5x^2+2)}\,dx$

$= \int_0^3 \frac{1}{2}\,dx + \int_0^3 \frac{10x}{10x^2+4}\,dx$ $\quad U = 10x^2+4 \quad du = 20x\,dx$

$dx = \frac{du}{20x}$

$= \boxed{\frac{3}{2} + \frac{1}{2}\ln\left(\frac{47}{2}\right)}$

Ex $\int e^x(4+e^x)^3\,dx$ $\quad U = 4+e^x \quad du = e^x\,dx$

$dx = \frac{du}{e^x}$

$\int U^3\,dx$

$\frac{U^4}{4} + C$

$\boxed{\frac{(4+e^x)^4}{4} + C}$

15 Feb 2010

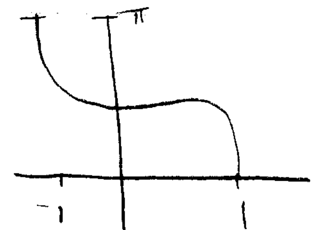
Inverse Trig Functions

$(\tan^{-1}x)' = \frac{1}{1+x^2}$

$(\sin^{-1}x)' = \frac{1}{\sqrt{1-x^2}}$

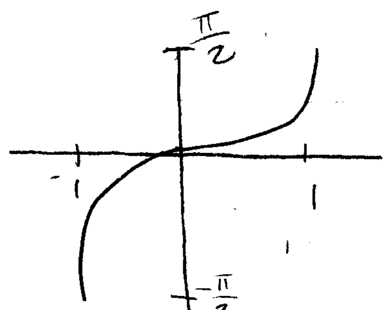
$(\sec^{-1}x)' = \frac{1}{x\sqrt{x^2-1}}$

$\cos^{-1} x$



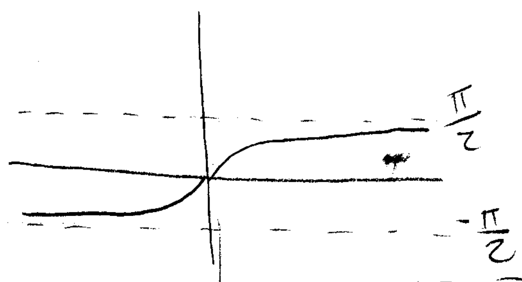
$\cos^{-1} x =$ "the" angle θ whose cos in x: the θ in $[0, 2\pi]$ so $\cos(\theta) = x$

$\sin^{-1} x$



The θ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ give every value of sin so $\sin\theta = x$

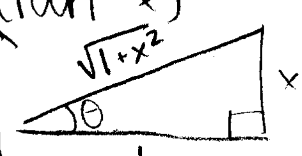
$\tan^{-1} x$



θ is $(-\frac{\pi}{2}, \frac{\pi}{2})$ so $\tan\theta = x$

Ex: $\cos(\tan^{-1} x)$

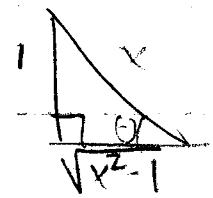
tan = $\frac{\text{opp}}{\text{adj.}}$



cos = $\frac{\text{adj}}{\text{hyp.}}$ $\boxed{\frac{1}{\sqrt{1+x^2}}}$

$$\sec(\csc^{-1}x) = \boxed{\frac{x}{\sqrt{x^2-1}}}$$

$$\csc = \frac{1}{\sin} = \frac{1}{\frac{opp}{hyp}} = \frac{hyp}{opp}$$



$$\sin^{-1}(\sin\tfrac{3\pi}{4})$$

$\frac{3\pi}{4}$ isn't in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ so go to θ $\boxed{\frac{\pi}{4}}$ b/c both have side $\boxed{\frac{1}{\sqrt{2}}}$

Integration

$$\int \frac{dx}{1+x^2} = \tan^{-1}x + C \qquad \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x + C \qquad \int -\frac{dx}{1+x^2} = \cot^{-1}x + 1$$

$$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{6}{\sqrt{1-t^2}}\,dt = 6\sin^{-1}t\Big]_{1/2}^{\sqrt{3}/2}$$

$$6\sin^{-1}(\tfrac{\sqrt{3}}{2}) - 6\sin^{-1}(\tfrac{1}{2})$$

$$6(\tfrac{\pi}{3} - \tfrac{\pi}{6}) = \boxed{\pi}$$

$$\int \frac{1+x}{1+x^2}\,dx = \int \frac{dx}{1+x^2} + \int \frac{x\,dx}{1+x^2}$$

$u = 1+x^2$

$du = 2x\,dx$

$x\,dx = \frac{du}{2}$

$$\int \frac{\frac{du}{2}}{u}$$

$$\tan^{-1}x + \tfrac{1}{2}\ln|u| + C$$

$$\boxed{\tan^{-1}x + \tfrac{1}{2}\ln|1+x^2| + C}$$

$$\boxed{\tan^{-1}x + \tfrac{1}{2}\ln(1+x^2) + C}$$

$$\int \frac{dx}{\sqrt{1-x^2}\,\sin^{-1}x} \qquad u = \sin^{-1}x \qquad du = \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \int \frac{du}{u} = \ln|u| + C = \boxed{\ln|\sin^{-1}x| + C}$$

Review RLM 7.104
3-5 on sunday

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx$ a is a constant

- Introduce new variable $x^2 = a^2 \cdot u^2$
  set $x = au$
  $dx = a\,du$

$\int \frac{dx}{\sqrt{a^2-x^2}} : \frac{a\,du}{\sqrt{a^2-a^2u^2}}$

$\frac{a}{\sqrt{a^2}} \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u)+C$

$\boxed{\sin^{-1}\left(\frac{x}{a}\right)+C}$

$\sqrt{a^2}$ $=1$

Ex. $\int \frac{x}{1+x^4}\,dx = \int \frac{x\,dx}{1+(x^2)^2}$ $u=x^2$ $du=2x\,dx$
$\frac{du}{2} = x\,dx$

$\int \frac{\frac{1}{2}du}{1+u^2} = \frac{1}{2}\tan^{-1}u + C$

$= \boxed{\frac{1}{2}\tan^{-1}(x^2)+C}$

16 Feb. 2010

$\int_0^1 \frac{1}{1+x^2}\,dx$ $\arctan x\Big]_0^1$

$\arctan(1) - \arctan(0)$

$\frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$

SOHCAHTOA

$\tan^{-1}(\sqrt{3}) = \boxed{\frac{\pi}{3}}$

$\cos(\arcsin \frac{1}{2})$

$\cos\left(\frac{\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{2}}$

$\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

$\tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$ in our range

$\tan^{-1}\left(\tan\left(\frac{5\pi}{6}\right)\right)$

$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \boxed{-\frac{\pi}{6}}$