Lecture 13

17 Feb 2010

Integration By Parts (Ch 8.1)

Product rule:

$$\frac{d}{dx}\left[f(x)\,g(x)\right] = f'(x)\,g(x) + f(x)\,g'(x)$$

Take $\int$ of both sides:

$$f(x)\,g(x) = \int f'(x)\,g(x)\,dx + \int f(x)\,g'(x)\,dx$$

$$\Rightarrow \quad \int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$$

Or: write

$u = f(x) \qquad v = g(x)$

$du = f'(x)\,dx \qquad dv = g'(x)\,dx$

$$\Rightarrow \quad \int u\,dv = uv - \int v\,du$$

Ex Find $\int x\sin(x)\,dx$.

Say $u = x$ $\quad v = -\cos x$

$du = dx$ $\quad dv = \sin(x)\,dx$

Then $\int x\sin(x)\,dx$

$$= \int u\,dv$$

$$= uv - \int v\,du$$

$$= -x\cos x - \int(-\cos x)\,dx$$

$$= \underline{\underline{-x\cos x + \sin x + C}}$$

Ex Find $\int \ln x\,dx$

Try $u = \ln x$ $\quad v = x$

$du = \frac{1}{x}dx$ $\quad dv = dx$

$$\int \ln x\,dx = \int u\,dv$$

$$= uv - \int v\,du = x\ln x - \int x\cdot\frac{1}{x}\,dx$$

$$= x\ln x - \int 1\,dx$$

$$= \underline{\underline{x\ln x - x + C}}$$

Ex Find $\int e^t t^2\, dt$.

Suppose we try $u = e^t$ $\quad v = \frac{1}{3}t^3$

$du = e^t\, dt \quad dv = t^2\, dt$

Then $\int e^t t^2\, dt = \int u\, dv = uv - \int v\, du$

$= e^t \cdot \frac{1}{3}t^3 - \int \frac{1}{3}t^3 e^t\, dt$

$\Longrightarrow$ getting harder! Wrong choice of u, v.

Take $u = t^2 \quad v = e^t$

$du = 2t\, dt \quad dv = e^t\, dt$

$$\int e^t t^2\, dt = \int u\, dv = uv - \int v\, du$$

$$= t^2 e^t - \int e^t\, 2t\, dt$$

Use int. by parts again: new u, v

$u = 2t \quad v = e^t$

$du = 2\, dt \quad dv = e^t\, dt$

So the original $\int$ becomes

$$= t^2 e^t - \left[\int u\, dv\right]$$

$$= t^2 e^t - \left[uv - \int v\, du\right]$$

$$= t^2 e^t - \left[2te^t - \int e^t\, 2\, dt\right]$$

$$= \underline{\underline{t^2 e^t - 2te^t + 2e^t + C}} \qquad = \underline{\underline{e^t(t^2 - 2t + 2) + C}}$$

Ex $\int_0^{\pi} t \sin 3t \, dt$

Pick $u = t \qquad v = -\frac{1}{3}\cos 3t$

$du = dt \qquad dv = \sin 3t \, dt$

$$\int_0^{\pi} t \sin 3t \, dt = \int_0^{\pi} u \, dv = uv \Big|_0^{\pi} - \int_0^{\pi} v \, du$$

$$= (t)\left(-\frac{1}{3}\cos 3t\right)\Big|_0^{\pi} - \int_0^{\pi} \left(-\frac{1}{3}\cos 3t\right) dt$$

$$= (t)\left(-\frac{1}{3}\cos 3t\right)\Big|_0^{\pi} + \frac{1}{3}\left(\frac{1}{3}\sin 3t \Big|_0^{\pi}\right)$$

$$= \left(-\frac{\pi}{3}\cos 3\pi - 0\right) + \frac{1}{9}(0 - 0)$$

$$= \underline{\underline{\frac{\pi}{3}}}$$

<u>Ex</u> $\int e^x \sin x \, dx$

Try $u = e^x$ $\quad v = -\cos x$

$du = e^x dx$ $\quad dv = \sin x \, dx$

$$\int e^x \sin x \, dx = \int u \, dv = uv - \int v \, du$$

$$= -e^x \cos x - \int (-\cos x) e^x \, dx$$

$$= -e^x \cos x + \int e^x \cos x \, dx$$

Int. by parts <u>again</u>:

$u = e^x$ $\quad v = \sin x$

$du = e^x dx$ $\quad dv = \cos x \, dx$

$$\int e^x \sin x \, dx = -e^x \cos x + \int u \, dv$$

$$\int e^x \sin x \, dx = -e^x \cos x + \left[uv - \int v \, du \right]$$

$$\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \int \sin x \, e^x \, dx$$

$$2 \int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x \, dx = \underline{\underline{\tfrac{1}{2}\left(-e^x \cos x + e^x \sin x\right) + C}}$$