Lecture 15

22 Feb 2010

[My web page: www.ma.utexas.edu/users/neitzke]

Exam WEL 1.316 7-9pm

EXAM REVIEW:

Int. by parts:

$$\int_0^1 16x^2 e^{2x}\,dx = \int_0^1 u\,dv$$

$u = 16x^2 \quad v = \frac{1}{2}e^{2x}$

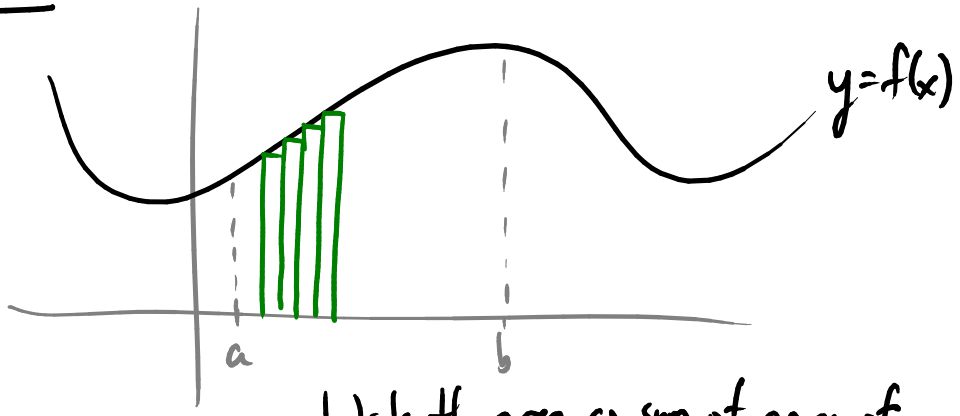
$du = 32x\,dx \quad dv = e^{2x}\,dx$

$$\int_0^1 u\,dv = uv\Big|_0^1 - \int_0^1 v\,du$$

$$= 8x^2 e^{2x}\Big|_0^1 - \int_0^1 \frac{1}{2} e^{2x}\, 32x\,dx$$

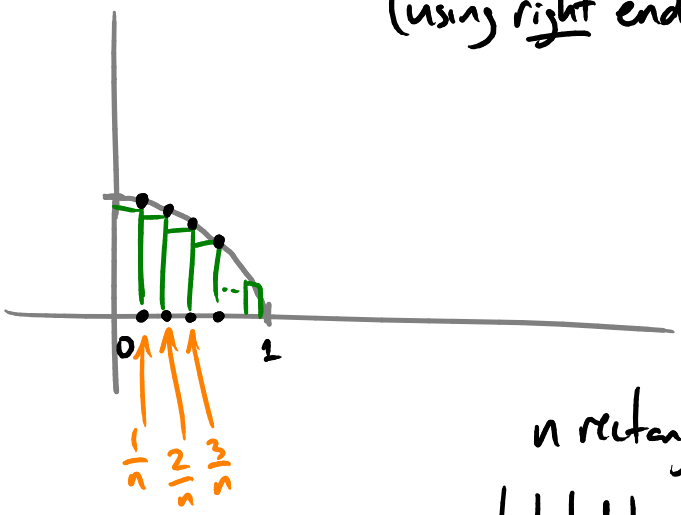
then do $\int$ by parts again...

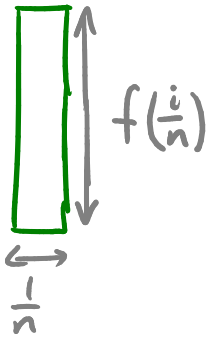
Riemann sums



Write the area as sum of areas of n little rectangles, then take $\lim_{n\to\infty}$ of the sum.

Ex Write $\int_0^1 (1-x^3)\,dx$ as a limit of Riemann sums (using right endpoints)





n rectangles

labeled by $i = 1, \dots, n$

$$\text{area} = \frac{1}{n} f\left(\frac{i}{n}\right) = \frac{1}{n}\left(1-\left(\frac{i}{n}\right)^3\right)$$

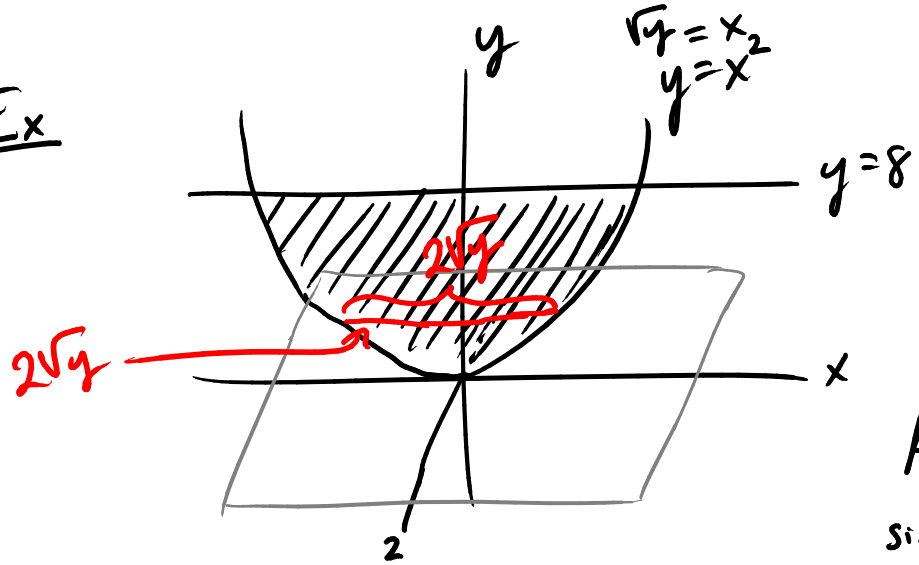
So the Riemann sum is

$$A = \sum_{i=1}^{n} \frac{1}{n}\left(1-\left(\frac{i}{n}\right)^3\right)$$

$$\left[\frac{i}{n} = a + i\Delta x\right]$$

Ex $\int \frac{\sin 2x}{\sin x}\,dx = ?$ Use $\sin 2x = 2\sin x \cos x$

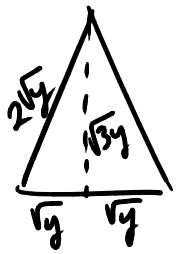
Ex



$$V = \int_0^8 dy\, A(y)$$

At fixed y, equilateral $\triangle$

side length $= 2\sqrt{y}$

$A(y) =$ 

$\frac{1}{2}bh = \frac{1}{2}(2\sqrt{y})(\sqrt{3}y)$

$= \sqrt{3} \cdot y$

$$V = \int_0^6 \sqrt{3} \cdot y \, dy = \underline{32\sqrt{3}}$$

FTC 1:

Ex If $F(x) = \int_5^x 3\sqrt{1+t^2}\, dt$

what is $F'(x)$?

$$F'(x) = \frac{d}{dx}\int_5^x 3\sqrt{1+t^2}\, dt = \underline{3\sqrt{1+x^2}}$$

Ex If $F(x) = \int_5^{x^2} 3\sqrt{1+t^2}\, dt$

what is $F'(x)$?

[from the chain rule: $\frac{d}{dx} x^2$]

$$F'(x) = \frac{d}{dx}\int_5^{x^2} 3\sqrt{1+t^2}\, dt = 2x \cdot 3\sqrt{1+x^4}$$

$$= \underline{6x\sqrt{1+x^4}}$$

Warnings:

$$\int \frac{1}{t^{1/3}}\,dt \neq \ln(t^{1/3})$$

$$\int t^{-1/3}\,dt = \frac{3}{2}t^{2/3} + C$$

$$\int \frac{1}{t}\,dt = \ln|t|$$

$$\ln(a) + \ln(b) = \ln(ab)$$

$$-\ln(a) = \ln\left(\frac{1}{a}\right)$$

$$b\ln(a) = \ln(a^b)$$

e.g. $$\ln(9) - 2\ln(6) = \ln(9) - \ln(36)$$

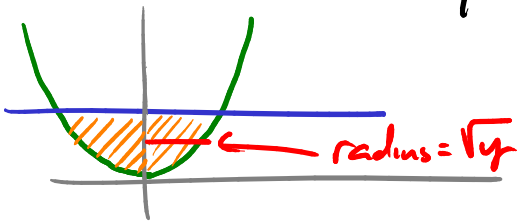
$$= \ln\left(\frac{9}{36}\right)$$

$$= \ln\left(\frac{1}{4}\right)$$

$$= -\ln 4 = -2\ln 2$$

Volumes of revolution

Calc the volume obt. by rotating the region bounded by $y = x^2$, $y = 1$ around the y-axis.



Cross-sections are circles area $A(y) = \pi r^2$

$= \pi (\sqrt{y})^2$

$= \pi y$

$$V = \int_0^1 A(y)\,dy = \int_0^1 \pi y\,dy = \frac{\pi}{2}$$

Ex $I = \int \frac{1}{2}\sin(\ln x)\,dx = ?$

Int by parts: $u = \sin(\ln x)$ . . .

. . $dv = dx$

$$\int e^x \sin x\,dx = (\text{stuff}) - \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = (\text{stuff})$$

$$\int e^x \sin x\,dx = \frac{1}{2}(\text{stuff})$$