Lecture 26 | 25 Mar 2010

Housekeeping: Guest lectures next MW (Maria Gualdani)

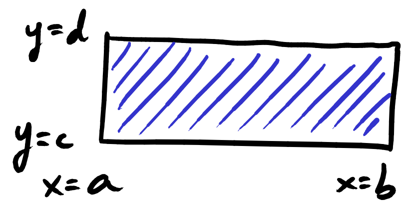
No office hrs next M

Extra office hr next F (11-12 in add^n to usual 10-11)

Exam 2 April 6

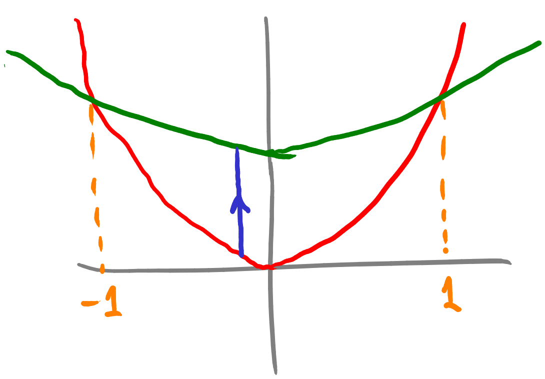
Last time: <u>iterated integrals</u> $\int_c^d \left[\int_a^b f(x,y)\, dx \right] dy$

= double integrals over <u>rectangles</u>



How about more complicated domains than rectangles?

<u>Ex</u> Evaluate $\iint_D (x+2y)\, dA$ where D is the domain lying between the curves $y=2x^2$ and $y=1+x^2$.



It's given by an iterated integral

$$\int_{-1}^{1} \left[\int_{2x^2}^{1+x^2} (x+2y)\, dy \right] dx$$

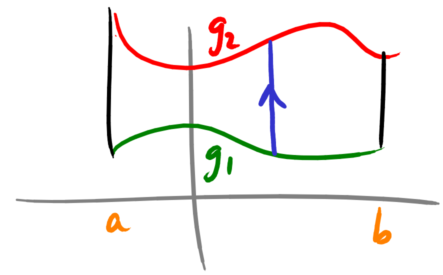
$$= \int_{-1}^{1} \left[xy + y^2 \Big|_{y=2x^2}^{y=1+x^2} \right] dx$$

$$= \int_{-1}^{1} \left[x(1+x^2) + (1+x^2)^2 - \left(x(2x^2) + (2x^2)^2 \right) \right] dx$$

$$= \cdots = \frac{32}{15}$$

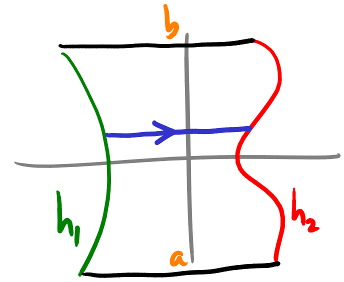
We use this basic method <u>whenever</u> we have a domain of the form

$$D = \{(x,y) : \ a \le x \le b, \ \ g_1(x) \le y \le g_2(x)\}$$



$$\iint_D f(x,y)\, dA = \int_a^b \left[\int_{g_1(x)}^{g_2(x)} f(x,y)\, dy \right] dx$$

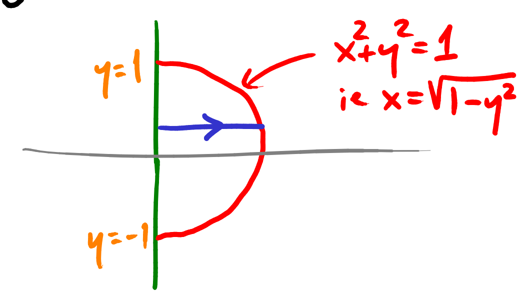
If $D = \{(x,y) : \ a \le y \le b, \ \ h_1(y) \le x \le h_2(y)\}$



$$\iint_D f(x,y)\, dA = \int_a^b \left[\int_{h_1(y)}^{h_2(y)} f(x,y)\, dx \right] dy$$

Ex $\displaystyle\iint_D xy^2\, dA$

D is the domain enclosed by
the line $x=0$
the circle $x^2+y^2=1$
with $x>0$



$$= \int_{-1}^{1} \left[\int_0^{\sqrt{1-y^2}} xy^2\, dx \right] dy$$

$$= \int_{-1}^{1} \left[y^2 \frac{x^2}{2} \Bigg|_{x=0}^{x=\sqrt{1-y^2}} \right] dy$$

$$= \int_{-1}^{1} \left[\tfrac{1}{2} y^2 (1-y^2) - 0 \right] dy = \cdots = \frac{2}{15}$$

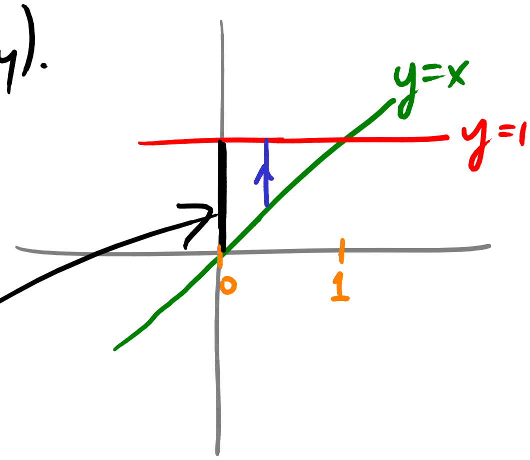
Ex $\displaystyle\int_0^1 \left[\int_x^1 \sin(y^2)\,dy\right] dx$

This looks hard (can't do the inside $\int$ over y).

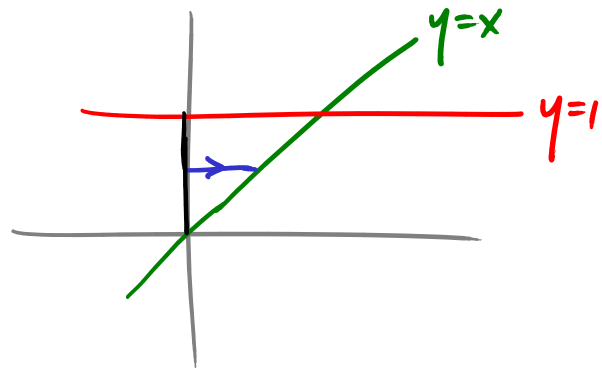
Interpret it as a double $\int$:

$$\iint_D \sin(y^2)\,dA$$

where D is this triangle



Do it by integrating over x first:



$$\int_0^1 \left[\int_0^y \sin(y^2)\,dx\right] dy$$

$$= \int_0^1 \left[x\sin(y^2)\Big|_{x=0}^{x=y}\right] dy$$

$$= \int_0^1 y\sin(y^2)\,dy$$

$$= -\frac{1}{2}\cos(y^2)\Big|_{y=0}^{y=1}$$

$$= \underline{\underline{\frac{1}{2}(1-\cos(1))}}$$