Lecture 5

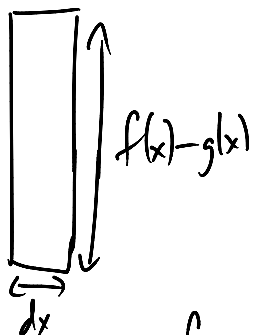
Admin: survey at tinyurl.com/yd9stt3w

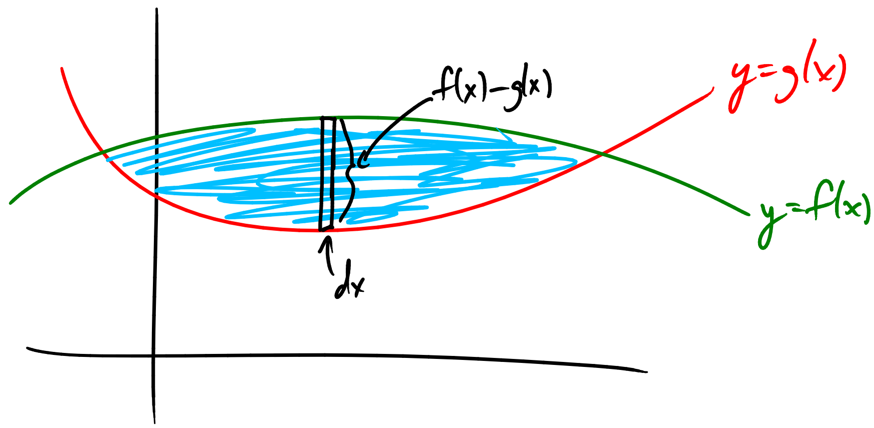
next LM Sat midnight (then M, W, Sat, ...)

HW Tue 3am

my office hr: Mon 2–3, Thu 5–6, RLM 9.134

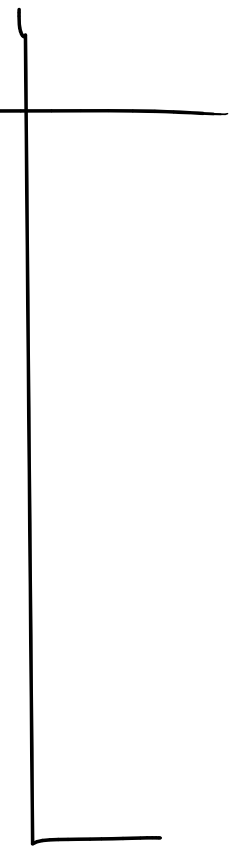
Last time: Area between curves





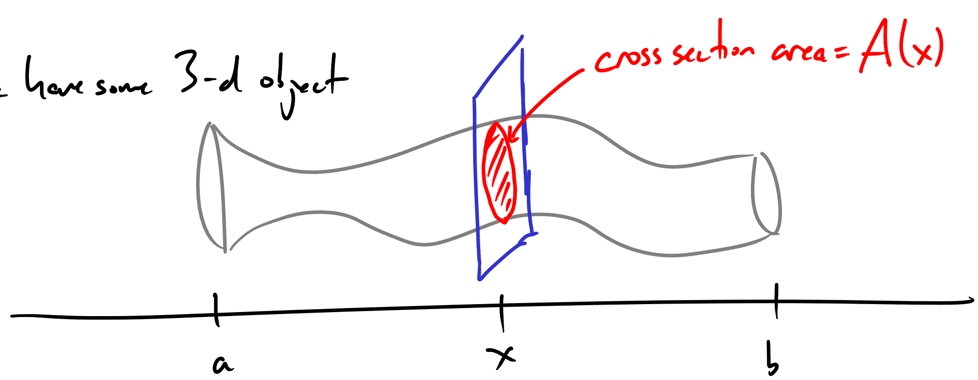
area of rect $= (f(x)-g(x))\,dx$

total area $= \int (f(x)-g(x))\,dx$



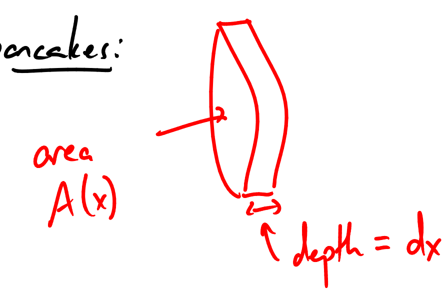
Volumes

Suppose have some 3-d object



Chop the object into slices that look like pancakes:

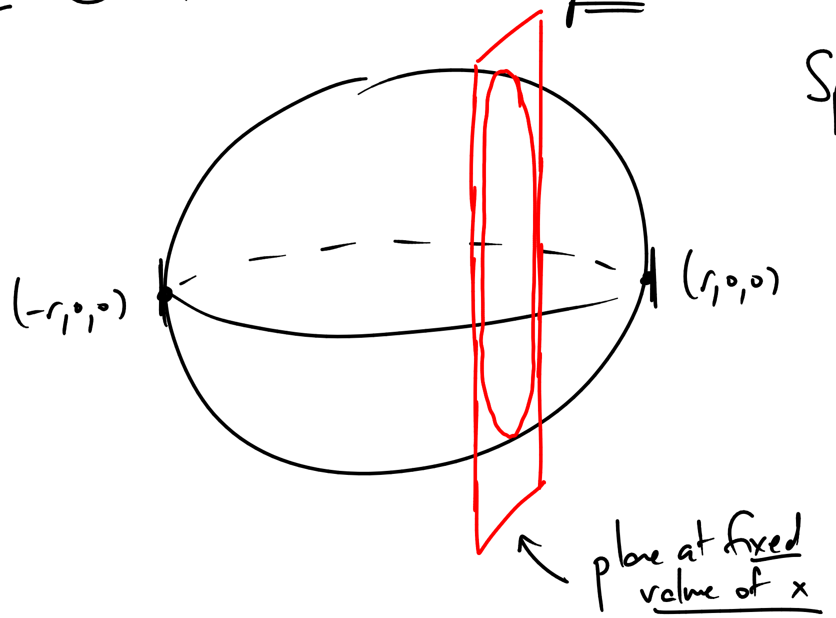
volume of the pancake $= A(x)\,dx$



To get the whole volume, add up the slices:

$$V = \int_a^b A(x)\,dx.$$

Q Calculate the volume of a sphere of radius r. (by slicing)



Sphere is $x^2+y^2+z^2=r^2$

$$V=\int A(x)\,dx$$

what is $A(x)$?

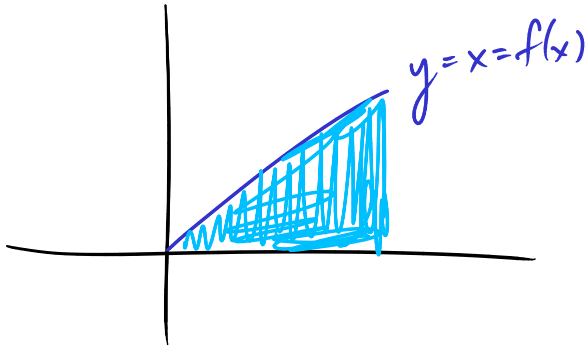
treating x as const.

$$x^2+y^2+z^2=r^2$$
$$y^2+z^2=r^2-x^2 \quad \text{circle of radius } \sqrt{r^2-x^2}$$
$$\text{area } A(x)=\pi\left(\sqrt{r^2-x^2}\right)^2$$
$$=\pi(r^2-x^2)$$

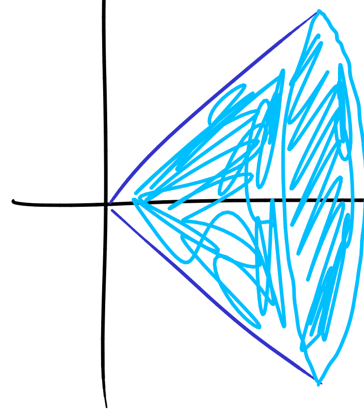
$$V=\int A(x)\,dx$$
$$=\int_{-r}^{r}\pi(r^2-x^2)\,dx$$
$$=\pi\left(r^2x-\tfrac{1}{3}x^3\right)\Big|_{-r}^{r}$$
$$=\pi\left(r^3-\tfrac{1}{3}r^3\right)-\left(-r^3+\tfrac{1}{3}r^3\right)$$
$$=\underline{\underline{\tfrac{4}{3}\pi r^3}}$$

A common type of solid: "solid of revolution" — take the region under some graph and revolve it around, say, the x-axis.

Ex



revolve $\rightsquigarrow$

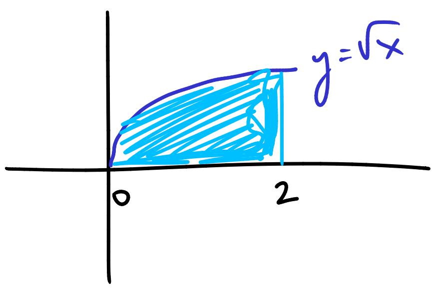


cone

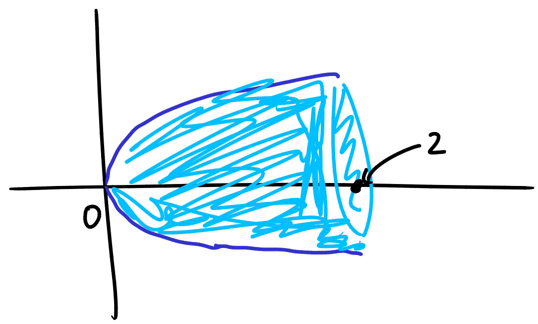
at fixed x,
Cross section: circle of radius $f(x)$

Cross-section area: $A(x) = \pi f(x)^2$

Q Find the volume of a solid obtained by revolving the area under $y = \sqrt{x}$ around the x-axis, with x from 0 to 2.



$\rightsquigarrow$



$$V = \int_0^2 dx\, A(x) = \int_0^2 dx\, \pi (\sqrt{x})^2 = \int_0^2 dx\, \pi x = \cdots = 2\pi$$

Could also revolve around y-axis:

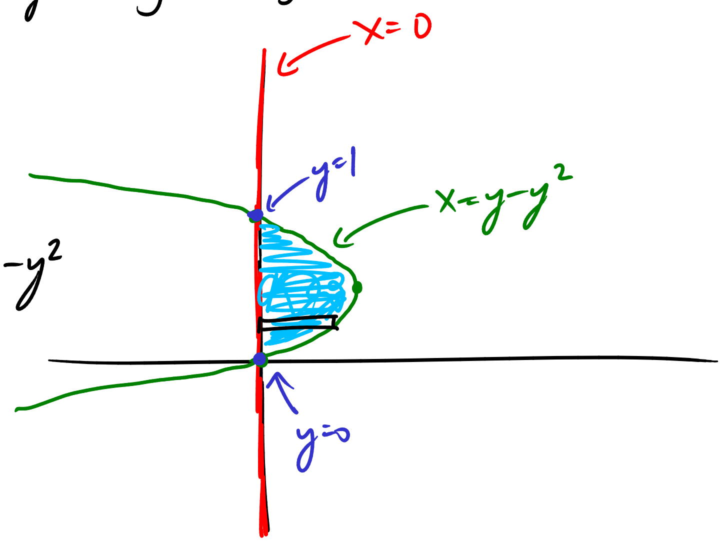
Ex Find vol. of region obtained by revolving the region between

$x = y - y^2$
$x = 0$

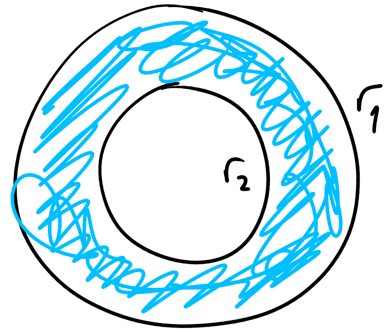
around the y-axis.
cross section: circle, radius $y - y^2$

$$V = \int_0^1 \pi (y - y^2)^2\, dy$$

$$= \cdots = \frac{\pi}{30}$$



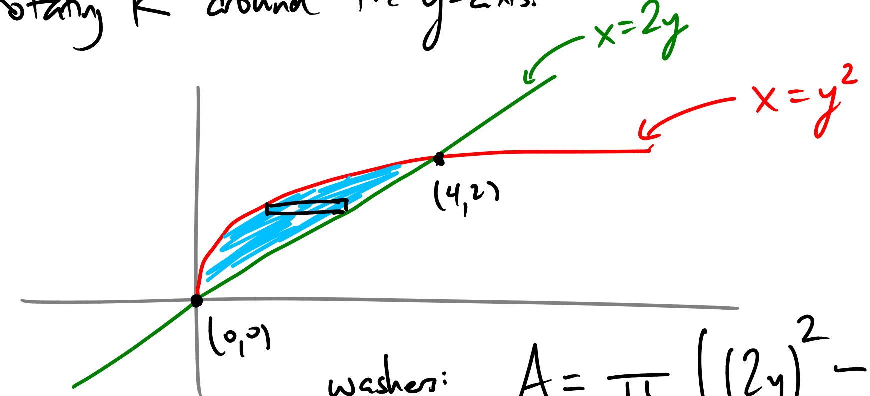
Another possibility: cross sections are "washers"



$$A = \pi(r_1^2 - r_2^2)$$

Q Let R be the region between $y=\sqrt{x}$ and $x = 2y$.

Find the vol. of solid obt by rotating R around the y-axis.



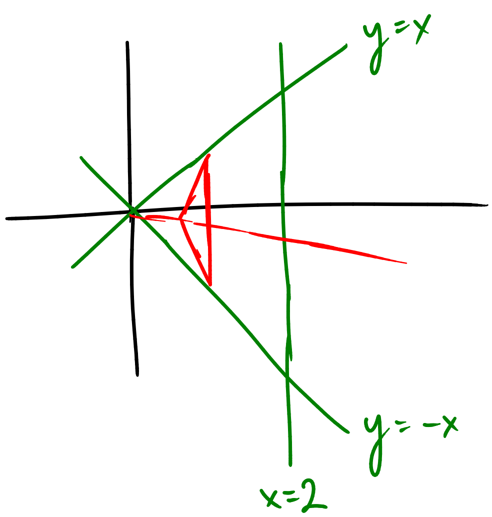
$$A = \frac{64\pi}{15}$$

washers: $A = \pi\left((2y)^2 - (y^2)^2\right)$

$= \pi(4y^2 - y^4)$

$$V = \int_0^2 \pi(4y^2 - y^4)\,dy = \cdots = \frac{64\pi}{15}$$

Ex Calculate the volume of a solid whose base is the region between $y=x$, $y=-x$ and $x=2$, and whose cross sections at fixed x are equilateral triangles.



$$\left(\frac{L}{2}\right)^2 + h^2 = L^2$$

$$h^2 = \frac{3}{4}L^2$$

$$h = \frac{\sqrt{3}}{2}L$$

$$A = \frac{1}{2}bh = \frac{1}{2}L\cdot\frac{\sqrt{3}}{2}L = \frac{\sqrt{3}}{4}L^2$$

$$V = \int_0^2 A(x)\,dx = \int_0^2 \frac{\sqrt{3}}{4}(2x)^2\,dx = \int_0^2 \sqrt{3}\,x^2\,dx = \cdots = \frac{8}{3}\sqrt{3}$$