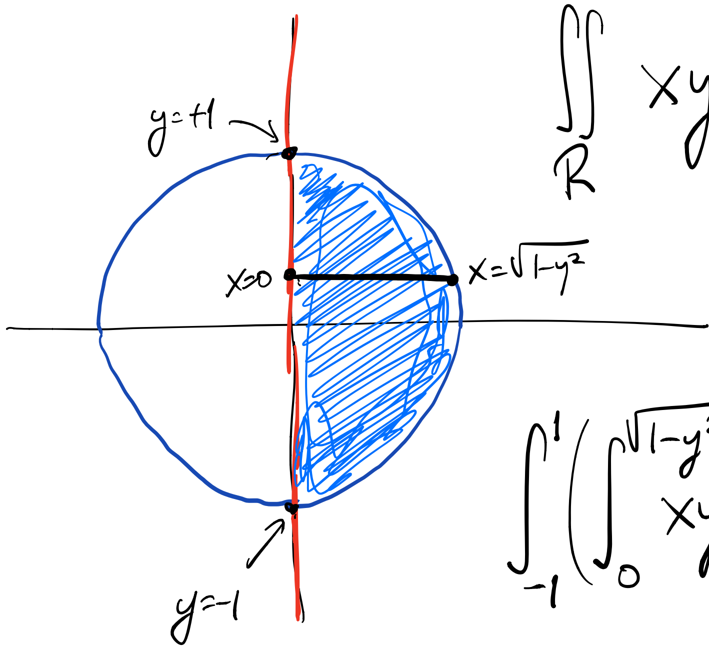


$$\iint_R xy^2\, dA$$

$$\int_{-1}^{1}\left(\int_0^{\sqrt{1-y^2}} xy^2 \, dx\right) dy$$

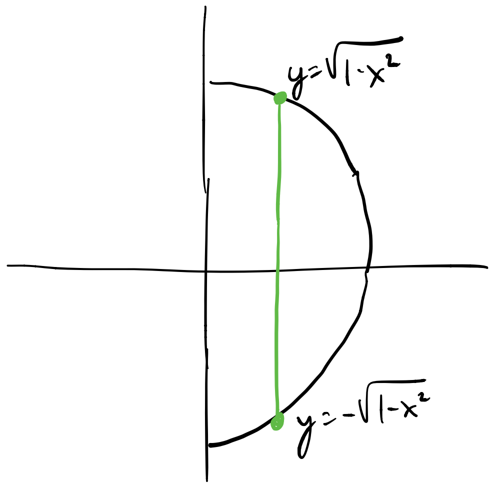
Inside integral: $\displaystyle\int_0^{\sqrt{1-y^2}} xy^2\, dx$

$$= \frac{1}{2}x^2y^2 \Big|_{x=0}^{x=\sqrt{1-y^2}}$$

$$= \frac{1}{2}(1-y^2)y^2$$

outside integral: $\displaystyle\int_{-1}^{1} \frac{1}{2}(1-y^2)y^2\, dy$

$$= \int_{-1}^{1} \frac{1}{2}y^2 - \frac{1}{2}y^4\, dy = \cdots = \underline{\underline{\frac{2}{15}}}$$



$$\int_0^1 \left(\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} xy^2 \; dy \right) dx$$

inside: $\left(\frac{1}{3} x y^3 \Big|_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} \right)$

$$= \frac{2}{3} x (1-x^2)^{3/2}$$

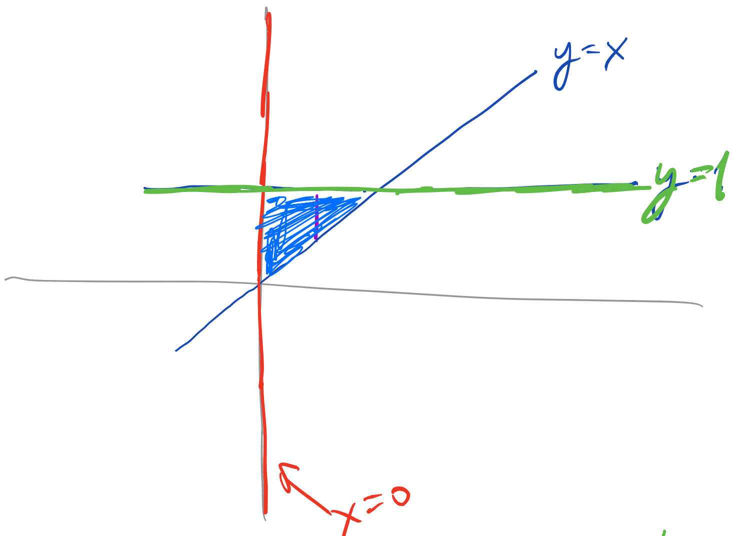
outside: $\frac{2}{3} \int_0^1 x (1-x^2)^{3/2} \, dx$

$u = 1-x^2$
$du = -2x \, dx$
$-\frac{1}{2} du = x \, dx$

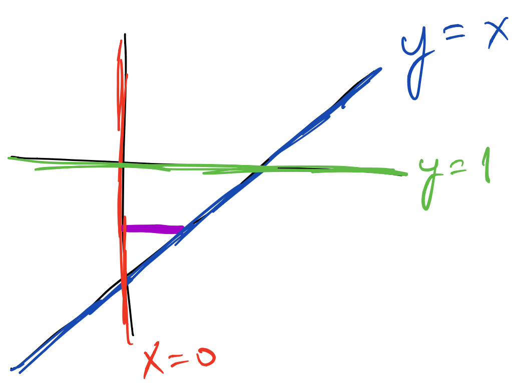
$$= \frac{2}{3} \int_1^0 u^{3/2} \left(-\frac{1}{2} du \right)$$

$$= -\frac{1}{3} \cdot \frac{2}{5} u^{5/2} \Big|_1^0 = \underline{\underline{\frac{2}{15}}}$$

Q $\displaystyle\int_0^1 \left[\int_x^1 \sin(y^2)\,dy \right] dx$



Try to rewrite this as $\displaystyle\int_0^1 \left(\int_0^y \sin(y^2)\,dx \right) dy$



inside integral: $\displaystyle\int_0^y \sin(y^2)\,dx = x\sin(y^2)\Big|_{x=0}^{x=y}$

$= y\sin(y^2)$

outside integral: $\int_0^1 y \sin(y^2)\, dy$

$$= -\frac{1}{2}\cos(y^2)\Big|_0^1$$

$$= -\frac{1}{2}\left(\cos(1) - 1\right)$$

Q1). Compute $\iint_R y \, dA$ R bounded by $y = x - 2$, $x = y^2$

2) Compute

$$\int_0^1 \int_{\sin^{-1}(y)}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\, dx\, dy$$

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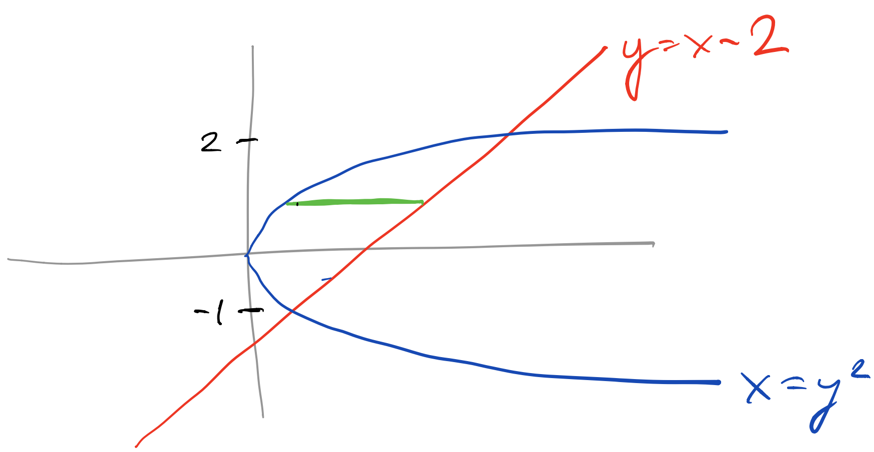
1)



$$\int_{-1}^{2}\left(\int_{y^2}^{y+2} y\, dx\right) dy$$

$$= \cdots = \underline{\underline{\frac{9}{4}}}$$

<u>Midterm 2</u> Tue Oct 31 7-9pm Jester A121A

first draft has <u>16 problems</u>

Sections 7.2, 7.3, 7.4, 7.5, 7.8, 14.3

- 7.2 — trig $\int$
- 7.3 — trig sub
- 7.4 — part frac
- 7.5 — strategy for $\int$
- 7.8 — improper $\int$
- 14.3 — partial deriv.

$$\int \frac{dx}{\sqrt{4x-x^2}}$$ <u>completing the square</u> then trig sub

$$\int \frac{x^2}{(16-x^2)^{3/2}}\, dx$$ trig sub $x = 4\sin\theta$

$$\int \frac{x}{(16-x^2)^{3/2}}\,dx$$

u-sub
$u = 16 - x^2$
$du = 2x\,dx$

$$\int \tan^{-1} x\,dx$$

by parts: $u = \tan^{-1} x$ $\quad v = x$
$du = \frac{1}{1+x^2}$ $\quad dv = dx$

$$x\tan^{-1}x - \int \frac{x}{1+x^2}\,dx$$

u-sub..

---

which converges?

① $\displaystyle\int_3^{\infty} \sin x\,dx$

② $\displaystyle\int_{17}^{\infty} \frac{1}{\sqrt{x}}\,dx$

③ $\displaystyle\int_3^{\infty} \frac{1}{x^3+1}\,dx$ $\quad$ ④ $\displaystyle\int_0^{4} \frac{1}{x}\,dx$

①: $\displaystyle\lim_{t\to\infty} \int_3^{t} \sin x\,dx = \lim_{t\to\infty} -\cos t + \cos 3$

DNE

<u>divergent</u>

(2) $\int_{17}^{\infty} \frac{1}{\sqrt{x}}\,dx$ divergent by the power rule: $\frac{1}{x^{1/2}}$ $\frac{1}{2}<1$

$\left(\int_{0}^{17} \frac{1}{\sqrt{x}}\,dx \right.$ convergent by power rule: $\frac{1}{x^{1/2}}$ $\left. \frac{1}{2}<1 \right)$

(3) $\int_{5}^{\infty} \frac{1}{x^3+1}\,dx$ convergent

because $\frac{1}{x^3+1} < \frac{1}{x^3}$

and $\int_{5}^{\infty} \frac{1}{x^3}$ is convergent by power rule: $3>1$

Q What is $\int_{0}^{2} \frac{1}{x^{1/3}}\,dx$?

$$= \lim_{t \to 0} \int_{t}^{2} \frac{1}{x^{1/3}}\,dx$$

$$= \lim_{t \to 0} \frac{3}{2} x^{2/3} \Big|_{t}^{2}$$

$$= \underline{\underline{\frac{3}{2} 2^{2/3}}}$$

$$\int_{x=7}^{x=8} \frac{\sqrt{x-7}}{x-3}\,dx$$

$u = \sqrt{x-7}$

$u^2 = x-7 \qquad x = u^2+7$

$2u\,du = dx$

$$= \int_{u=0}^{u=1} \frac{u \cdot 2u\,du}{u^2+4}$$

$$= \int_0^1 \frac{2u^2\,du}{u^2+4}$$

$$\begin{array}{r|l} & 2 \\ \hline u^2+4 & 2u^2+0u+0 \\ & 2u^2 \quad\quad +8 \\ \hline & -8 \end{array}$$

$$= \int_0^1 2 - \frac{8}{u^2+4}\,du$$

$$= \int_0^{1/2} 2 - \frac{8}{4t^2+4} \cdot 2dt \qquad \begin{array}{l} u = 2t \\ du = 2dt \end{array}$$

$$= 1 - 4\tan^{-1}(t)\Big|_0^{1/2}$$

$$= 1 - 4 \cdot \tan^{-1}\left(\tfrac{1}{2}\right)$$