Problem 1. Find the multiplicative inverses of the following elements of $\mathbb{Z}_{13}$:
(a) $2$
(b) $5$
(c) $18$
(d) $144$

Problem 2. (a) Find the multiplicative inverse of $800$ in $\mathbb{Z}_{997}$.
(b) Solve the equation $800x + 45 = 0$ in $\mathbb{Z}_{997}$. (Your answer $x = \cdots$ should be the unique element of $\mathbb{Z}_{997}$ that solves this equation.)
(c) Solve the congruence $800x + 45 \equiv 0$ modulo 997. (The point is that the heart of the problem is that the phrasing of both question and answer are different from (b), even though the essential math is the same. This time, you must give me all the integers $x$ that solve this congruence. There are infinitely many.)

Problem 3. Find all solutions to the following linear congruences. You may use either form (b) or (c) from the previous problem—whichever you prefer. But the form of your answer must match the form of the question.
(a) $3x \equiv 2$ in $\mathbb{Z}_7$, or equivalently $3x \equiv 2 \mod 7$.
(b) $6x \equiv 3$ in $\mathbb{Z}_9$, or equivalently $6x \equiv 3 \mod 9$.
(c) $987x \equiv 610$ in $\mathbb{Z}_{1597}$, or equivalently $987x \equiv 610 \mod 1597$.

Problem 4. The following are not meant to be super hard, just help you avoid common errors.
(a) Construct integers $a$, $b$ and $c$ such that $a \mid bc$, but $a \nmid b$ and $a \nmid c$.
(b) Construct three integers $x$, $y$ and $z$ that have no common divisors $>1$ (meaning that no number larger than 1 divides all three), yet their pairwise gcd’s gcd$(x,y)$, gcd$(y,z)$ and gcd$(z,x)$ are all bigger than 1.

Problem 5. Suppose $n$ is a positive integer and write $a$ for $2^n - 1$.
(a) $a$ being prime requires $n$ to be prime. (Hint, that might or might not be helpful: think about geometric series.)
(b) The converse fails. That is, $n$ being prime does not force $a$ to be prime.