Find the intersection of two planes with equations (1A)

Plane #1: $(x-1)+2y+(z+1)=0$

Plane #2: $3(x-1)+4y-7(z+1)=0$

Method 1

By inspection / the form of these equations I see that $a=(1,0,-1)$ is a point in the intersection.

If v is the direction vector of my line, then since v lies in Plane #1, $v \perp (1,2,1)$ ← normal vector for plane 1

and since v lies in Plane #2, $v \perp (3,4,-7)$.

An easy way to get such a vector is to take

$$v=(1,2,1)\times(3,4,-7)=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 3 & 4 & -7 \end{vmatrix}$$

$$=(-14-4)\hat{i}-(-7-3)\hat{j}+(4-6)\hat{k}=-18\hat{i}+10\hat{j}-2\hat{k}.$$

So $\boxed{l(t)=(1,0,-1)+t(-18,10,-2)}$ is my line.

Method 2

I'm just going to solve this system of equations.

$$-2\big((x-1)+2y+(z+1)\big)=0$$
$$+\ 3(x-1)+4y-7(z+1)=0$$

$$(x-1)-9(z+1)=0$$

$$(x-1)=9(z+1)$$

$$\boxed{x=9z+10}$$

Substituting back in to the first equation (since it looks nicer to me), I get

$$(9z+10)-1\ \ +2y+\ z+1=0$$
$$10z+10+2y=0$$
$$\boxed{y=-5z-5}$$

So my line (taking $z=t$) has parametric eq'ns

$$\boxed{x=9t+10,\ y=-5t-5,\ z=t.}$$

1B

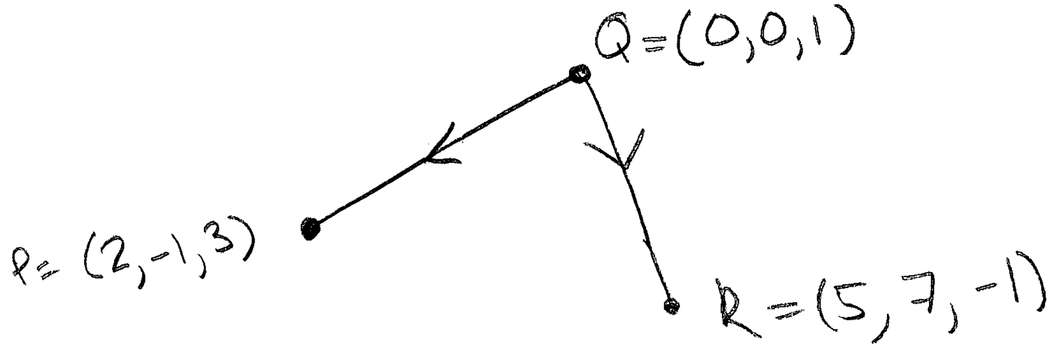
A line $\ell(t) = a + tv$ is perpendicular to $\langle 1, -2, 0 \rangle$ exactly when $v \perp \langle 1, -2, 0 \rangle$, which happens exactly when $v \cdot \langle 1, -2, 0 \rangle = 0$.

So, since

$$v \cdot \langle 1, -2, 0 \rangle = \langle 9, -5, 1 \rangle \cdot \langle 1, -2, 0 \rangle$$
$$= 9 + 10 = 19 \neq 0,$$

$\boxed{\text{NO}}$

(2) I want to find a normal vector to this plane



$$\overrightarrow{QP} = (2,-1,2)$$
$$\overrightarrow{QR} = (5,7,-2)$$

$\rightsquigarrow$ I can take $\vec{n} = \overrightarrow{QP} \times \overrightarrow{QR}$

$$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 2 \\ 5 & 7 & -2 \end{vmatrix} = (2-14)\hat{\imath} - (-4-10)\hat{\jmath} + (14+5)\hat{k}$$
$$= -12\hat{\imath} + 14\hat{\jmath} + 19\hat{k}$$

I'll use Q as my 'base point' (since it's simplest)

and I get

$$-12(x-0) + 14(y-0) + 19(z-1) = 0$$

or $\boxed{-12x + 14y + 19z = 19}$