

Sample

Name and Discussion Section Time: \_\_\_\_\_

M 408C

Exam 2

Reid Spring 2008

1. Consider the curve given by:

$$x^2 - 2xy + y^3 = 1$$

(a) Let  $P = (2, 1)$ . Show that  $P$  is a point on this curve. (5 points)

(b) Use implicit differentiation to find the equation of the tangent line to the curve at  $P$ . Where does the tangent line meet the x-axis? (15 points)

2. An angler is fishing from a bridge 30 feet above the water. He has a fish at the end of his fishing line, and starts reeling in the line at 3 feet per second. At what speed is the fish moving through the water when the amount of line out is 40 feet. Assume the fish is at the surface of the water. (15 points)

3. Find the real numbers  $a$ ,  $b$  and  $c$  that determine a cubic polynomial

$$f(x) = x^3 + ax^2 + bx + c$$

having critical numbers at  $x = 1$  and  $x = -2$ , and for which  $f(1) = 0$ . (15 points)

4. A rectangular box with a square base is to hold 125 cubic inches. Material for the sides costs \$2 per square inch and the material for the top and bottom costs \$1 per square inch. Find the dimensions of the most economical box. (15 points)

Questions 5 and 6 refer to the function

$$f(x) = \frac{x-2}{x^2}$$

5.(a) Identify the domain of  $f$ . Compute  $\lim_{x \rightarrow \pm\infty} f(x)$  and identify any horizontal and vertical asymptotes in the graph of  $f$ . (7 points)

(b) Compute  $f'(x)$  and find the critical numbers of  $f$ . (4 points)

(c) Find the intervals on which  $f$  is increasing/decreasing and classify any local extrema. (6 points)

6.(a) Determine the intervals on which  $f$  is concave up/down, and any inflection points of  $f$ . (10 points)

(b) Sketch the graph of  $f$ , marking clearly the  $x$  and  $y$  intercepts as well as the information you have already obtained. (8 points)