Name and Discussion Section Time:____________________

M 408C **FINAL** **Reid Sample**

1. A particle P moves along the x-axis with position function at time t being $s(t) = t^5 - 5t^3 + 10$.

(a) What is the velocity and acceleration at $t = 2$? **6 points**

(b) At what times is the particle stationary? **4 points**

(c) On which intervals is the particle moving to the right (resp. left). **5 points**

2. Find the general form of the anti-derivatives for the following functions.

(a) $\frac{e^x}{e^{2x}+2e^x+1}$ **10 points**

(b) $\frac{x}{\sqrt{25-x^2}}$ **10 points**

3. Answer **YES** or **NO** to the following. To get full credit **you must explain your answers and show your work**:

(a) Does the following limit exist: **4 points**

$$\lim_{x\to -1} \frac{3x^7+16x^3+x+6}{x^9+6x^4+4x^2+3}.$$

(b) Does the following limit exist: **6 points**

$$\lim_{x\to 0} \frac{3x^2}{2\tan^2 2x}.$$

(c) Is the function $f(x) = \begin{cases} e^x & x<0 \\ 2 & x=0 \\ e^{-x} & x>0. \end{cases}$ is continuous at 0? **4 points**

(d) Is the function $h(x) = \frac{x}{x+1}$ is one-to-one. **6 points**

4.(a) Show that the function $f(x) = \begin{cases} 1 - x & x < 1 \\ x^2 - 3x + 2 & x \geq 1. \end{cases}$ is differentiable at 1. **10 points**

(b) Let $f(x) = 2\sqrt{\pi - \arctan 2x}$ and $g(x) = \frac{x^2+1}{x+2}$. Compute $(f \circ g)'(0)$. **10 points**

5.(a) Sketch the region R bounded by the curve $x = 2y - y^2$, and the y-axis from $y = 0$ to $y = 2$. **8 points**

(b) Compute the area of the region R. **12 points**

6. Evaluate the definite integrals.

(a) $\int_1^2 (\frac{x}{2} - \frac{2}{x})dx$. **10 points**

(b) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$ **10 points**

7. A kite is flying at a height of 300 feet and moving in a horizontal wind. When 500 feet of string is out the kite is pulling the string out at a rate of 20 feet/sec. What is the kites (horizontal) velocity at this instant. **15 points**

8. Find the area of the largest rectangle that can be inscribed in the ellipse

$$\frac{x^2}{4} + \frac{y^2}{25} = 1. \qquad \textbf{15 points}$$

9.(a) Sketch the region Ω bounded by the curves $x = y^2 + 3$, and $x + y = 5$ marking carefully the points where these curves meet. **8 points**

(b) Find the volume of the solid obtained by revolving the region Ω (from part (a)) about **the y-axis**. **12 points**

Questions 10, 11 and 12 refer to the function

$$f(x) = \frac{x^2 - 5}{x + 3}.$$

10. (a) Find the intervals on which f is increasing and those on which it is decreasing. **10 points**

(b)Determine the critical values of f and the nature of any local extrema. **5 points**

11. Find the intervals on which f is concave up/down and any inflection points. **10 points**

12. Describe any horizontal and vertical asymptotes, x and y intercepts and then sketch the graph **clearly marking the information on the graph. 10 points**