From Artin

Chapter 7 (page 224) 7.2, 8.1, 8.3, 8.5.

Chapter 11 (page 354) 1.6, 1.7, 1.8.

Others:

1. Prove that there is no simple group of order 56.

2. Finish the proof that there are no non-abelian simple groups of order \( n \leq 100 \) with \( n \neq 60 \) (i.e. finish the product of three primes case).

3. Prove that if \( H \neq 1 \) is a normal subgroup of \( A_5 \) then \( H \) contains a 3-cycle.

4. Prove that if \( |G| = 60 \) and \( G \) contains more than one Sylow 5-subgroup then \( G \) is simple.

5. Prove that a group of order 108 has a normal subgroup of order 9 or 27.

6. (Tricky) Suppose that \( |G| = 231 = 3 \cdot 7 \cdot 11 \). Show that the Sylow 7 and 11 subgroups are normal and that the Sylows 11-subgroup is contained in \( Z(G) \).

7. Let \( G \) be a finite group, \( H \) a normal subgroup of \( G \) and \( P \) a Sylow \( p \)-subgroup of \( H \). Show that \( G = HN_G(P) \) and \([G : H]\) divides \(|N_G(P)|\). (This is known as Frattini’s argument.)

8. Let \( \mathbb{Z}[^\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\} \). Prove that this ring has infinitely many units.

9. Let \( R \) be a commutative ring with 1, and let \( R[x] = \{\text{polynomials with coefficients in } R\} \).
   (a) Prove that \( R[x] \) is a commutative ring with 1.
   (b) Prove that if \( R \) is an integral domain, then \( R[x] \) is an integral domain.
   (c) What are the units in \( R[x] \)?