From Artin
Chapter 11 (pages 354–357) 3.2, 3.5, 3.8 (just take \( R = \mathbb{F}_p \)), 3.9, 3.12, 3.13, 7.5.

Others:
1. Let \( R = \mathbb{F}_2[x] \) and \( p(x) \in R \) given by \( x^2 + x + 1 \). Prove that \( R/ < p(x) > \) is a field with four elements.
2. The center of a ring \( R \) (with 1) is:
\[
Z(R) = \{ z \in R : rz = zr \ \forall r \in R \}.
\]
   (a) Prove that \( Z(R) \) is a subring of \( R \) that contains the identity.
   (b) Prove that the center of a division ring is a field.
3. Let \( G \) be a finite group, and \( R \) a commutative ring with 1. Prove that \( Z(RG) \) is non-trivial. (Hint: sum all the elements of \( G \)).
4. Which of the following are ideals of \( \mathbb{Z}[x] \)?
   (a) the set of all polynomials with constant term a multiple of 3.
   (b) the set of all polynomials whose coefficient of \( x^2 \) is a multiple of 3.
   (c) the set of polynomials in which only even powers of \( x \) appear.
   (d) the set of polynomials \( p(x) \) such that \( p'(0) = 0 \) (where \( p'(x) \) is the usual derivative function).
5. Prove that \( M_2(\mathbb{R}) \) contains a subring isomorphic to \( \mathbb{C} \).
6. Let \( R \) be a commutative ring with 1.
   (a) Define
   \[
   \text{rad } I = \{ r \in R : r^n \in I \text{ for some } n \in \mathbb{N} \}.
   \]
   Prove that \( \text{rad } I \) is an ideal of \( R \) containing \( I \) (called the radical of \( I \)).
   (b) An element \( a \in R \) is called nilpotent if \( a^m = 0 \) for some \( m \geq 1 \). Prove that the set of nilpotent elements form an ideal (the nilradical), and that \( 1 + a \) is a unit.
   (c) What is \( \text{rad } I/I \)?
7. Determine the nilpotent elements of \( \mathbb{Z}/72\mathbb{Z} \).